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AN ANALYSIS OF THE "ELECTRIC FYDP" FORCE COSTING MODEL

By

George Leon Moses

United States
Naval Postgraduate School

THESIS

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George Leon Moses

Thesis Advisor: M. G. Sovereign

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An Analysis of the "Electric FYDP"

Force Costing Model

by

George Leon Moses
Major, United States Army
B.S., United States Military Academy, 1963

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ABSTRACT

Analysis of the "Electric FYDP" force costing model's ability to estimate alternative force structure costs is presented and related to the current Department of Defense planning, programming, and budgeting system. The "Electric FYDP" cost model is described in economic and mathematical terms. An analysis of the model's workability, as defined by Michio Hatanaka, in modeling a military service economy for use in force cost prediction and force cost analysis is presented. The problems in estimating the parameters of the "Electric FYDP" are discussed and current parameter estimation procedures are appraised. Preliminary sensitivity analysis is performed. The conceptual use of estimates of force cost variance in scheduling procurement of future weapons systems with a binding budgetary constraint is discussed.

TABLE OF CONTENTS

TABLE OF ABBREVIATIONS

ADP	- Automatic Data Processing
DoD	- Department of Defense
FYDP	- Five Year Defense Plan
I-O	- Input-Output
JSOP	- Joint Strategic Operations Plan
OSD	- Office of Secretary of Defense
OSD(SA)	- Office of Secretary of Defense (System Analysis)
POM	- Program Objective Memorandum
PPBS	- Planning, Programming, and Budgeting System
RDT and E	- Research, Development, Test, and Evaluation
TFGM	- Tentative Fiscal Guidance Memorandum
TOA	- Total Obligational Authority

I. INTRODUCTION

In 1970 the Department of Defense presented an automated force costing model to assist in apportioning the DoD budget among the various DoD agencies. The model is called the "Electric FYDP" force costing model. This paper concerns that model and is written in order to appraise it.

Two methods may be used to evaluate the applicability of a mathematical model. One method involves the use of past data to generate predictions from the model and compare such predictions with experience. Repeated testing in this retrospective manner can permit confident validation of the model's relationships. A second method is to evaluate the logic of the model. This method entails an explicit identification and evaluation of the assumptions which are required by the logical relationships of the model. An analysis of the logic of input-output models in general and the "Electric FYDP" force costing model in particular will be presented in this paper.

The first section of this paper is intended to provide the context within which the "Electric FYDP" costing model is used. That section will describe the planning, programming, and budgeting system currently employed at DoD and will accentuate the important role the "Electric FYDP" cost model serves in that process.

One use of cost predictions is to provide information for the scheduling of new weapon systems. The scheduling

of new weapon systems is constrained by force cost estimates. In order to show this relation, the problem of force modernization scheduling is presented. However, the scheduling problem will be described only. Conceptual ideas are presented which may be relevant to its solution, but no attempt will be made to offer a solution. It is included to point the way for future research and show the scheduling problem's dependency upon reliable force costing procedures.

In Section III the "Electric FYDP" force costing model is described in economic and mathematical terms. The process by which the model is used to predict force costs and to perform force cost analysis is presented in Section IV. The purpose of that section is to explain how the "Electric FYDP" cost model operates.

The necessity for the validation of a mathematical model is to allow the user of the model to feel confident that the model computes accurate force costs where accuracy is as will be described in the body of this report. Until some validation procedure is used to determine the validity of the model the user can place little confidence in the model. Model validation can be performed by either empirical testing and comparison of predictions and realizations, or by an examination of the model's ability to accurately describe the process being modeled. In the case of the "Electric FYDP" model, empirical testing validation procedures were not done because the necessary data was not available. Hence,

an analysis of the "Electric FYDP" force cost model's logical structure is presented.

Section V presents an analysis of the workability of the "Electric FYDP" cost model for use in force cost prediction and force cost analysis. Michio Hatanaka's scheme of analysis is used [6]. The intention of the analysis is to evaluate all implied and explicit assumptions required by the "Electric FYDP" force costing model. In the author's opinion, this scheme of model validation does not appear to have been performed prior to the use of the "Electric FYDP" model at DoD. The preliminary evaluation presented in Section V and the resulting conclusions indicate that thorough validation procedures using past data to test and analyze the predictive ability of the model should be performed.

In Section VI the problem of estimating the model parameters is examined. The current method of estimation of parameters using one-sample point is explained, and the implication of using one-sample point estimates of variables which are subject to random variations is discussed. Significant variations in total force cost estimates due to parameter errors are demonstrated by use of a simplified version of the model which uses the same procedures employed by the "Electric FYDP" force cost model.

The "Electric FYDP" costing model is a deterministic costing model. Currently, the estimates of model parameters are computed as though they were not subject to random variations. Analysis of the model's logic presented in Section IV

implies that the estimates of the model parameters are subject to error. It is the thesis of this paper that the "Electric FYDP" costing model can better serve its intended purpose if the errors in its parameter estimates were accounted for in its costing methodology. It is explained in Section VI that the use to be made of the model dictates the parameter estimation methodology which sould be used. The difference between describing causality and describing correlation is discussed. To explain causality requires descriptive parameters based upon logical relationships; whereas, to explain correlation requires a predictive parameter based upon regression theory. The distinction may seem trivial. If the model is used to describe only, estimating the expected value of individual random variables becomes the estimation problem. If the model is to be used to predict, then simultaneous multiple equation regression analysis is the estimation technique which should be used. It is not necessarily true that predictive parameters will be the same as descriptive parameters for input-output models. Aggregation errors alone may be sufficient to make them incomparable. If the DoD decision maker has little confidence in the parameters of the model, he will have little confidence in the solutions produced by the model.

II. CONCEPTUAL DESCRIPTIONS OF PROBLEMS TO BE ADDRESSED

A. THE PLANNING, PROGRAMMING, AND BUDGETING SYSTEM

Current trends in the Federal Government indicate that a reordering of national priorities is taking place [1]. As a result of this reordering, it is the author's opinion that the Department of Defense will not receive the degree of funding it has been accustomed to receiving. As Department of Defense appropriations decrease in real terms, competition for funds among all Department of Defense agencies will heighten.

Quantitative analysis will play a more influential role in the budgetary process. In addition to political factors and threat implications, the decision maker in defense management will be forced to rely more heavily on quantitative analysis to assist in deciding difficult fundamental intra-service decisions regarding budgeting of total Department of Defense funds [2].

A system of planning, programming, and budgeting has been used in national defense management. Planning and budgeting should be effectively linked to insure that force structures planned to meet national strategy are provided by sufficient appropriations. This concept has been widely accepted as desirable since the early 1960's. There has been great difficulty in obtaining effective application. Past problems with the PPBS in the Department of Defense seem to be related to a lack of timely effective communications between OSD(SA) and the DoD agencies. The current

PPBS at DoD is designed to eliminate this lack of timely effective communication between OSD and the DoD agencies [3].

It has been suggested that the current PPBS will require more and better analysis of possible alternatives by each DoD agency competing for funds. Early in the budgeting cycle a tentative fiscal guidance memorandum is issued to each DoD agency by the OSD staff. The TFGM tells each agency the funding it may expect over the next five years by FYDP program category. Contained in each TFGM are dictated expenditure levels called "fences." "Fences" are amounts that must be budgeted by the agency concerned for specified programs or groups of programs. Each military service and other defense agencies respond to the TFGM by compiling and submitting to OSD(SA) a document called the Program Objective Memorandum. In the POM each agency and service proposes three alternative programs in terms of forces, manpower, and dollar costs. The first alternative total program reflects a base program in which each service and agency has budgeted total TFGM funds while observing all "fences." The second alternative total program reflects a program in which the agency or service concerned has budgeted total TFGM funds, i.e., "fences" are not observed. The third alternative total program is known as the "decremented case." It provides the agency's or service's total program if the total funds specified in the TFGM were to be decremented by a specified amount both with and without "fences."

The forces contained in the POM submission reflect the Joint Strategic Operations Plan I, the Strategy Guidance

Memorandum, and the Joint Force Memorandum. These documents are formed in the earlier planning phase of the budgeting cycle in which a strategy is planned and forces required are determined by the National Security Council, Secretary of Defense, and Joint Chiefs of Staff.

Within the POM there is a written assessment of the risks involved or the military gains accrued as a result of the variances between alternative program allocations of the total budget. This assessment of risks and military gains will be done by the agency or service submitting the POM. The burden of detailed force planning and trade-off analysis is placed with the services and agencies. This increased analysis role for the services and DoD agencies will require a marked increase in the analysis capability of all DoD agencies [2].

As with earlier versions of the PPBS at DoD level, the new PPBS will require effective timely communication between each service and the OSD(SA) staff. The budget variances between the TFGM and POM must be resolved before each service's budget is finally set. This will require rapid definition of policy issues and the resolution of differences in methodology early in the budgeting cycle.

B. THE "ELECTRIC FYDP" FORCE COSTING MODEL

An automated total force costing model would assist the speed and clarity of the dialoque between OSD(SA) and the services. Use of ADP requires a total force costing model that can be computer coded. Without the use of an automated

total force costing model, analysis required in composing the POM and in performing analysis of alternative force structures and alternative budgets must be done by hand calculations using very crude estimating procedures that adversely effect the reproducibility of the analysis [3]. Manual computation of the numerous alternative POM force structure creates a severe burden for service staffs. The POM alternatives must be computed in a rapid turn around time and the size of the service staff used for cost computation is limited.

It is important to note the distinction between force analysis and force cost analysis. Force analysis is analysis by which an optimal force "mix" is selected. Here optimal usually means the maximum of a criterion of effectiveness. Force cost analysis is analysis which delineates feasible force structure levels constrained by a budget. These are two different but highly related problems. Force analysis determines the force levels and force "mix" required to achieve a planning objective. Force Cost analysis determines what force levels and "mixes" may be achieved when constrained by a budget ceiling.

A computerized force costing model has been put into use at DoD. It is called the "Electric FYDP System" and its purpose is to facilitate the dialoque between OSD and the military services [3]. The use of the "Electric FYDP System" force costing model has permitted OSD to explicitly state the costing procedures used in setting the TFGM for each DoD agency and service. This step is necessary for

efficient and rapid convergence to policy and methodology issues that must be resolved prior to the end of the budgeting cycle. Use of the "Electric FYDP System" permits all parties involved to explicitly identify the factors and assumptions underlying the TFGM. It has also been proposed as a tool for equal cost trade-off analysis of alternative force structures.

The "Electric FYDP System" is a deterministic model. It provides solutions as though they are not subject to random variations in the model parameters. A deterministic force costing model has several disadvantages. It treats a force cost estimate as a deterministic variable. In this author's opinion, force structure cost estimates are subject to random variations due to random variations in the model's parameters.

Failure to treat force cost estimates as subject to random variations affects scheduling of force modernization. Estimates of RDT and E, investment, and operation costs to support a force structure are used as budget ceilings on the total of life-cycle cost streams for all new weapon systems. The total life-cycle cost stream for all of force modernization must not exceed the total of the estimates of RDT and E, investment, and operation budgets. If they do, production schedules for new systems must be altered or funds must be transferred from one account to another. The problem of scheduling force modernization and its dependency upon total force cost estimates will be shown later in this section.

The force modernization scheduling problem will be discussed only. No attempt will be made to solve the problem except in a conceptual sense. The ideas presented are this author's ideas about how the scheduling problem could be approached. The purpose for including its discussion in this paper is to illustrate how a force costing model, which accounts for random variations in its estimates, relates to the modernization scheduling problem.

For the remainder of this paper the author will use examples relating to the Army. In the author's opinion, problems experienced by the Army in the force costing field are probably representative of the problems experienced by the other services.

The Army receives tentative fiscal guidance via the TFGM which provides, by fiscal guidance categories for each year of the current FYDP, a breakout of the Army's anticipated budget. An example of the Army's anticipated budget breakout (without actual dollar entries) as contained in the TFGM is shown in Table I. Such a table exists in the TFGM for each year of the FYDP currently being proposed. The figures which actually would appear in the TFGM tables are total obligated authority to the nearest tenth of a billion dollars. The TFGM is the first attempt to meet strategy outlined in the JSOP while at the same time observing the total DoD budget as anticipated year by year for the coming five years. Note that the entries which would appear in Table I give TOA for a one year period. Graphically, the

Army's TFGM for all five years can be displayed as in Figure 1.

The variability of the accounts which comprise the TFGM is important for two reasons. The first reason is related to total force costing. The total force cost can be divided into that cost which is directly attributable to the force structure and that which is indirectly attributable to the force structure. The indirect costs of the force structure are costs such as RDT and E, training, base operating support, personnel management, and other indirect support function costs. It seems to the author that the indirect cost portion of total force cost has the greatest flexibility for a given force structure, because when total cost of a given force structure must be reduced it is more likely that indirect support costs can be reduced without reducing force structures. Reductions in direct costs of the force structure are difficult without reducing force structure levels. The second way in which variations in TFGM accounts is important is related to force modernization scheduling. In the scheduling of force modernization, variations of expenditure rates in the TFGM categories affect the selection of a schedule for phasing a new system into the force structures. The RDT and E, investment, and operation accounts for the TFGM constrain the total life cost streams of all systems to be phased into future force structures. This will be illustrated in the discussion of force modernization scheduling.

TABLE OF CATEGORIES FOR TENTATIVE FISCAL

GUIDANCE FOR THE ARMY AS CONTAINED IN THE TFGM

FY 72 FISCAL GUIDANCE

TOA in FY 70 (xx) Billions)

	Army
Major Mission Forces	
Strategic Forces	-
Sub-total strategic forces	-
Land Forces	-
Mobility Forces	-
Total Major Mission Forces	-
Other Missions	
Intelligence and Security	-
Communications	-
Research and Development	-
Support to other Nations	-
Total Other Missions	-
General Support	
Bases and Individual Support	-
Training	-
Command	-
Logistics	-
Total General Support	-
Miscellaneous Costs	
Retired Pay Appropriations	-
Family Housing Program	-
Military Construction	-
Total Miscellaneous Costs	-
GRAND TOTALS	-

Table I

Note that each block in Figure 1 represents the total obligated funds for a major fiscal guidance category for a given year of the TFGM. Figure 1 represents major categories only, but the resolution to sub-categories can easily be done. Figure 1 displays only one alternative expenditure <u>rate</u> for each major categroy during one year. All that the TFGM requires is that TOA for each element not be exceeded each year. Figure 2 shows four alternative expenditure rates for a single category of the TFGM during one year. For each alternative, the total expenditure is the area beneath the dashed curve and the TOA is the area beneath the solid curve. There exists an infinite number of such alternatives for each category. For each alternative depicted in Figure 2, expenditures do not exceed TOA. A similar set of alternative expenditure rates can be constructed for each fiscal guidance category.

The rate of expenditure during any one year period will show little variation for some accounts. The yearly expenditure in the account called "Bases and Individual Support" cannot be varied a great deal because radical changes in yearly expenditure rates in this category would be intolerable. For instance, military pay accounts are strongly related to manpower levels. Radical shifts in expenditure rates in this account without like changes in manpower levels would create shortages in funds to pay personnel. For a given manpower level, the estimate of the "Bases and Individual Support" account cost estimate would show only slight variation.

Graphic Summary of Army's

TFGM for AU Five Years by

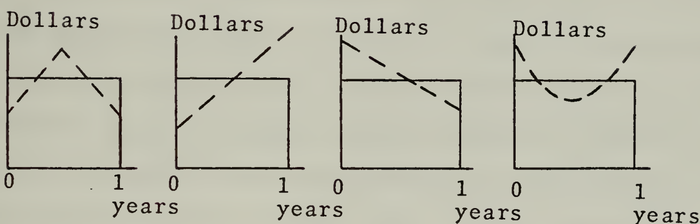
Major Fiscal Guidance Category

1. Major Mission Force Costs.
2. Other Mission Force Costs.
3. General Support Costs.
4. Miscellaneous Costs.

Figure 1

Four Alternative Expenditure Rates

for One Account for One Year of TFGM



Solid Line Denotes TOA Level

Dashed Line Denotes Expenditure Rate Alternative

Figure 2

This invariability does not apply for some accounts. The "Research and Development" account and system procurement sub-accounts are two such TOA accounts whose expenditure rates can exhibit great variability. These activities are not principally dependent upon force structure levels. In the author's opinion, TOA for these accounts are greatly dependent upon threat analysis, the state of the technological base, political climate, and many other highly uncertain factors. The Research and Development effort for the coming five years is not determined by the predicted size of the force structure, rather, it is more likely to be determined by predicted requirements for new systems which is in turn dependent upon the highly variable factors cited previously. The procurement of new weapon systems is also highly variable. The decision of what to buy, when to buy, and how much to buy are strongly related to the urgency of the requirement and the development state of alternative weapon systems. These factors themselves are extremely variable for several reasons. The degree of urgency is dependent upon when the need for a new weapon system is greatest. The time when the need is greatest may change as a result of changes in the nature of the threat, enemy capabilities, technological developments, and a host of other factors. The state of development is related to technological uncertainities which can be extremely variable. It is the author's opinion that some fiscal guidance category accounts are highly related to force size and invariable while others, such as "Research

So far costing of force structures and budgeting has been discussed in general. Costing of total force structures in order to determine the total obligational authority for each of the TFGM accounts is performed using a costing model. The accuracy of the costing model used is important to the force modernization scheduling problem. Future weapon system development, procurement, and operating costs must be scheduled. The schedule is dependent upon the estimated budgets for development, procurement, and operating costs. If the budgets are inaccurately estimated by the cost model used, force modernization schedules may be rendered infeasible. The following discussion relates to this problem. Conceptual ideas which might be used to account for inaccuracies of the cost model estimates of budgets and their effect on scheduling are presented.

C. FORCE MODERNIZATION SCHEDULING

In addition to costing alternative force structures and performing equal cost trade-off analysis, there exists the vital problem of scheduling the Army's equipment modernization program. As our perception of national threats and a potential enemy's capability change over time, new weapons systems must be introduced into the Army's force structure and obsolete systems retired to keep abreast of technological changes in weapons systems. This is nothing more than taking advantage of technological opportunities available or developed. The choice of which system to buy and when to buy it is difficult to make in the face of a

fiscal constraint that has been predicted and set by an earlier appropriations decision. With the constraint of a decreasing real value of the Army budget, a method must be devised to provide the best phasing of new systems into the Army force structure according to some criterion. Desired operational capability events for new systems are based upon strategy and future force structures. The criterion used to select from among alternative schedules should be based upon strategy and future force structures. The criterion used to select from among alternative schedules should be based upon these events. (A possible criterion will be proposed in a following paragraph.) It is important for the Army decision maker to know what feasible alternative phasing schedules are available to him before he can make a proper decision. Without an effective method for constructing and ordering alternative schedules, the Army may find itself examining a very limited set of alternatives which might not include the optimal schedule (optimal according to a criterion to be proposed). With a scheduling model that can formulate all feasible alternatives permitted by estimated budget ceilings, and can then order these alternatives according to a proper criterion, the best alternative can be selected.

The scheduling problem could be solved by a scheduling model that would associate a degree of error with each alternative schedule. Cost streams for system RDT and E, investment, and operating costs can be constructed with

confidence intervals [4]. Budget allocations for RDT and E, investment, and operating cost accounts can be projected with confidence intervals. When examining alternative schedules for phase-in of a new system in the face of a binding budgetary constraint, it is imperative that the degree of overlap between the confidence interval of the cost stream and the budget confidence interval be known. To illustrate the importance of this concept consider the following example.

Suppose the projected Army budget accounts for RDT and E, investment, and operating costs over the next fifteen years is an indicated by the graph in Figure 3. Suppose that the 90% confidence interval for each account is given by the dashed line envelope for each account graph. Note that the total budget allocated for each category would be firm for the early years with increasing uncertainity associated with the out years. Suppose further that the Army is considering a modernization program that entails operation, investment, and RDT and E for two weapons systems. Call these two systems System A and System B. Further suppose that the time phasing of the costs associated with these two systems is as indicated in Figure 4. Note that these cost streams are estimates. The dashed line envelope about each estimate is the 90% confidence interval associated with each estimate. The life-cost streams are the predicted time and cost factors arrived at by systems analysts in conjunction with industry and military planners. Industry contributes

Fifteen Year Predicted Army RDT and E,

Operation, and Investment Accounts

with 90% Confidence Intervals

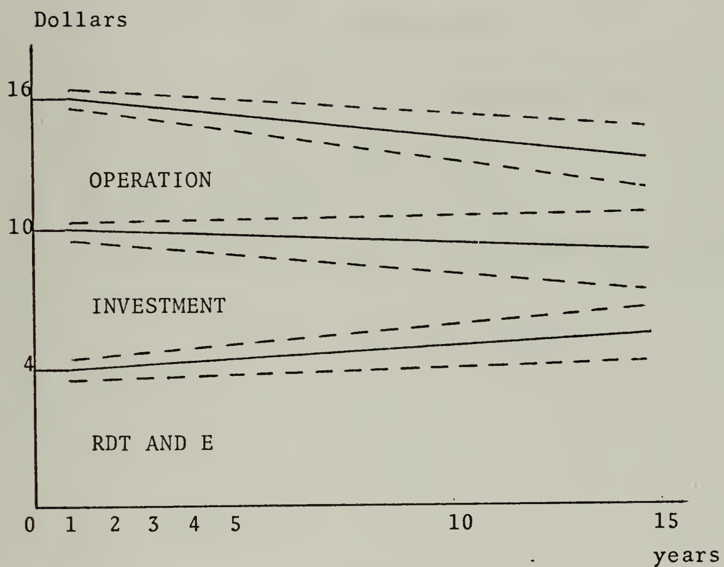
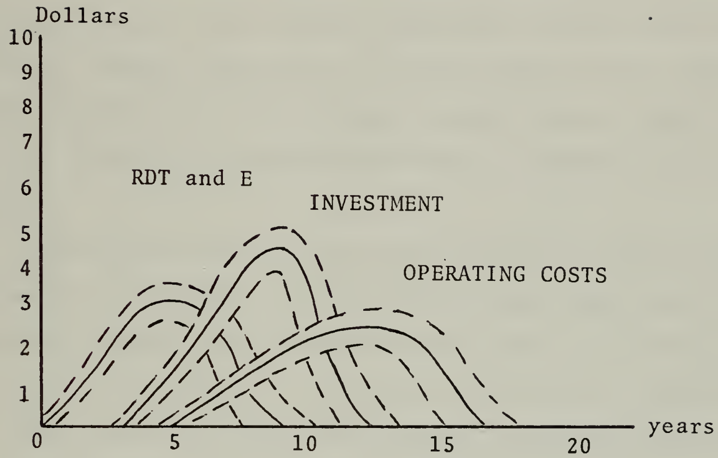
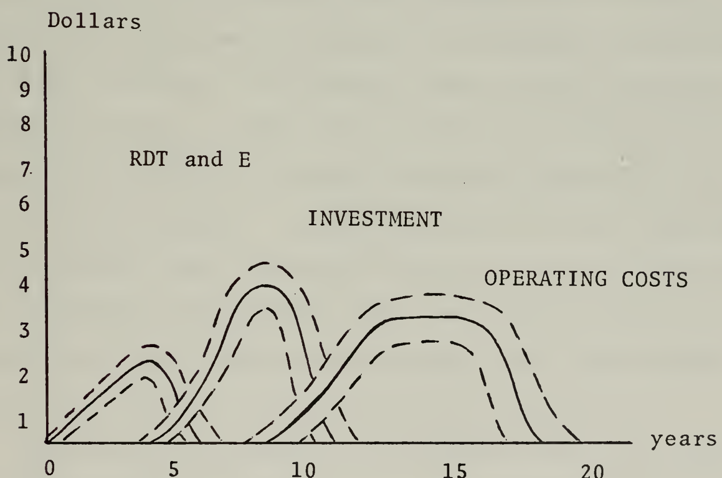


Figure 3

Life Cost Streams for Systems A and B with 90% Confidence Intervals



System A Life-Cost Stream



System B Life-Cost Stream

Figure 4

the time elements regarding RDT&E and production constrained by technology, expansion capabilities, and other production constraints. The military contribution to the life-cost schedules is the time phasing requirements based upon threat analysis, weapon technology, training requirements, and no budget constraint. The life-cost stream reflects what is desired by military planners and what is feasible for industry.

Associated with each life-cost stream are three events important to planning and budgeting. These operational capability events can provide the basis for a criterion to be used in ordering all alternative procurement schedules. The first event is called the initial operating capability. This event occurs when sufficient quantity of a new system is predicted to be available for procurement in order to permit the commencement of phase-in of the new system into the force structure. The second event is the full operational capability. This event occurs when sufficient procurement is predicted to enable complete phase-out of the system to be replaced. If the new system is not replacing an old system, full operation capability would be attained when predicted procurement allows the planned level of operation of the system to be reached. The third event occurs when the old system phase-out is completed.

The points in time when these three events are feasible and most desirable are variable. A system cumulative cost estimate at any given point in time is a variable. The expenditure rate is also variable because of the uncertainity

associated with the most desirable and feasible timing of the initial operating capability, full operational capability, and old system phase-out. The initial operating capability event time is dependent upon when full operational capability is planned. The planned time for full operational capability is dependent upon the urgency of the requirement, the state of development of complementary systems (for example, a life support system would be complementary to a high altitude bomber system), and other variable factors. Hence, the total RDT and E expenditures is variable from year to year and within each year due to the variability in the desired operational event times.

How do these life-cost streams fit into budget constraints? What is the best method to schedule both systems in order that operational events for both systems will occur when desired? For that alternative schedule which schedules both systems so that operational events occur as closely as possible to desired times, consistent with budget constraints, what is the degree of assurance that budgeted account levels will not be exceeded? These are the questions which a scheduling model should answer.

A criterion for ranking alternative system procurement schedules is the proximity of the operational event times of the alternative schedule to the currently desired operational event time consistent with budgetary constraints. Use of this criterion bridges the gap between what is feasible in a budgetary context and what is desired from the

planning viewpoint. An algorithm which can accomplish such a ranking of all alternative procurement schedules would prove highly valuable in military management.

Dr. George Patton has authored a doctoral thesis entitled Optimal Scheduling of Resource Constrained Projects. Interviews by the author with Dr. Patton reveals that Dr. Patton feels that the scheduling algorithm described in his paper can be applied to the force modernization scheduling problem. The feasibility and details of its application will not be discussed further in this paper, but it should certainly be researched as a scheduling algorithm.

What ways can life-cycle cost streams be changed to conform to the bounds of a budget constraint? First, the life-cost streams can be stretched over a longer time period. This will reduce the rate at which costs are increased by a given system. An element of the force structure can be phased out in an accelerated manner; thereby decreasing total force costs at a faster rate which insures conformity to a decreasing budget constraint.

Returning to the example of finding alternative schedules for phasing Systems A and B into the force structure subject to the budget constraint depicted in Figure 1, it is important to be aware of methods that could possibly be used to compute confidence intervals for budgets and cost-streams. Two methods could possibly be used. The first is nothing more than a manager's estimate based upon his prior judgement of the variance which he expects in the anticipated budget level. This could be stated as a percentage of the

anticipated budget. Or by use of regression analysis, total budgets could be predicted with confidence intervals. Using national economic variables as explanatory variables and total budget or sub-program budgets as the explained variable, samples from earlier budgets could be used to predict future budgets with confidence intervals. Similar methods could be used in predicting cost-stream confidence intervals by using appropriate explanatory variables.

To simplify the explanation of the example, only the RDT and E account is used to demonstrate the scheduling problem and conceptually illustrate an approach for optimal scheduling. The same ideas apply when there are n systems to be scheduled and m accounts. Figure 5 depicts the RDT and E account extracted from Figure 3. First year RDT and E requirements for System A can conceptually be derived by totaling the area beneath the RDT and E life-cost stream estimate for System A during the first year. The same can be done for each succeeding year. A 90% confidence interval for each year RDT and E requirement for System A can be derived by totaling the area beneath the upper limit of the RDT and E life-cost stream 90% confidence interval and likewise for the lower limit. Similar quantities for the first year RDT and E requirements for System B can be derived. In Figure 6 first year RDT and E requirements for System A and B are shown in (a) and (b) respectively. The total with its 90% confidence interval is shown in (c). As each successive yearly total RDT and E requirements for both systems

RDT and E Account Extracted from Figure 3

with 90% Confidence Interval

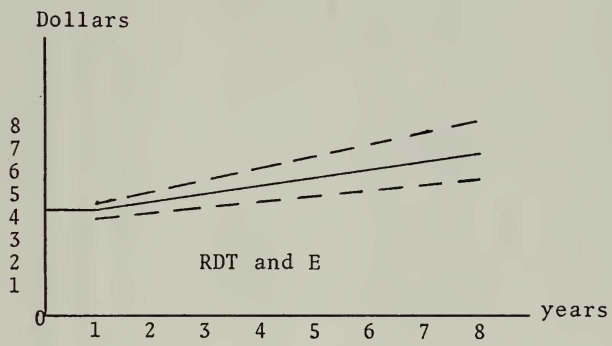


Figure 5

Individual and Combined First Year

RDT and E Requirements for Systems A and B

with 90% Confidence Intervals

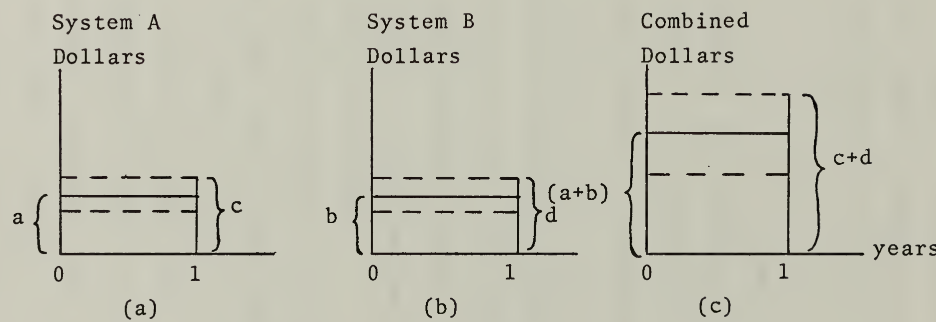


Figure 6

is derived, a point in time will be reached where the aggregate requirement confidence interval will intersect the RDT and E budget estimate confidence interval. When this intersection condition exists a degree of confidence indicating the risk of exceeding the budget estimate for RDT and E could conceptually be found. Such a condition is depicted in Figure 7.

By stretching one or both of the RDT and E cost streams a constant degree of certainity of remaining within budget levels during any time period can be maintained. However, by stretching a system cost-stream the operational capability events for that system will be delayed. This will decrease its military effectiveness by moving operational event times further away from desired event times. The problem then becomes that of deciding which system to stretch in order to minimize costs due to schedule slippage and minimize the effects of changed operational event times.

In Figure 7 note that from time point three to time point four the confidence intervals of the budget estimate and RDT and E combined requirements overlap. This indicates that a risk of RDT and E budget overruns exists if RDT and E life-cost streams as depicted in Figure 4 are used in the force modernization procurement schedule. The likelihood of budget overrun is conceptually a function of the amount and direction of overlap between the two confidence intervals. The decision as to which of the two system cost streams to change in order to decrease the likelihood of budget overrun seems to be a function of the total cost levels in RDT

RDT and E Budget Estimate and Combined RDT and E Requirements for System A and B Illustrating 90% Confidence Interval Overlap

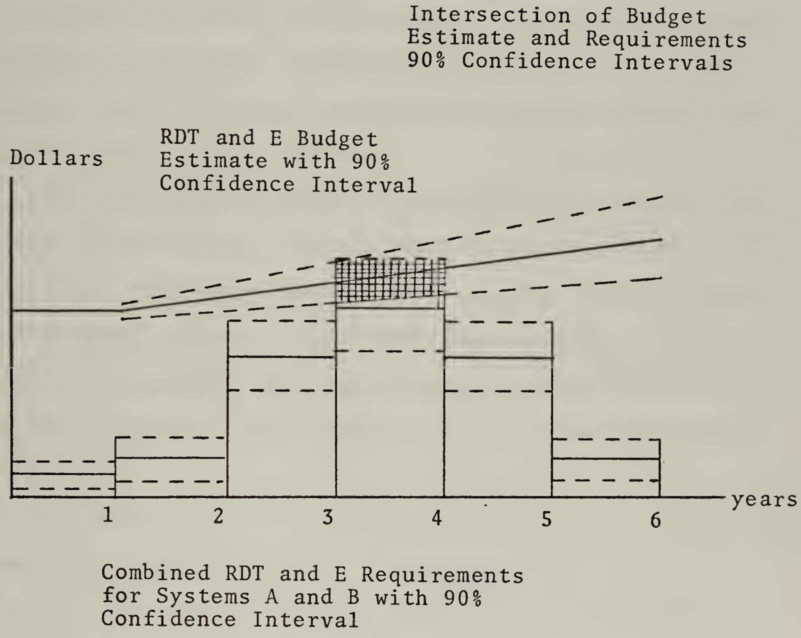


Figure 7

and E effort during the time interval (3,4) for each system and the cost rate for each system during the time period (3,4). Stretching either or both system RDT and E efforts will delay the stretched system's operational events which will decrement that system's military effectiveness in the force structure relative to the system it is to replace or relative to the some undelayed form of the system.

The scheduling of force modernization procurement will not be pursued further in this paper. A description of the problem was presented in order to point the way for future research and characterize how the scheduling problem is related to force cost estimates.

The remaining sections of this paper will address the "Electric FYDP" force costing model in order to answer three questions. What type of cost model is used in the "Electric FYDP System"? What if any underlying assumptions of such a model are violated? How sensitive are answers provided by the basic model to small changes in the parameters of the model?

The estimation of force structure costs is an important element in defense planning. Estimating the total cost of a force structure is necessary, but it is not as useful as estimates of the two component parts of total force structure costs. Total force cost is comprised of the direct costs of the men, material, and dollars consumed <u>directly</u> by the force structures and the men, material, and dollars consumed <u>indirectly</u> by the support establishment which supports the force structure. It has always been a difficult

task to attribute support establishment costs to the appropriate elements of the force structure. Input-output analysis lends itself to this function. By appropriate definition of support elements, I-O analysis can be used to estimate the flow of support costs between elements of the support establishment required to provide the support of a given force structure. I-O analysis can separate out the direct and indirect costs of a force structure; and subsequently properly attribute support costs to the appropriate elements of the force structure. In the next section the method by which the "Electric FYDP" basic model uses I-O analysis to identify direct and indirect costs will be explained.

III. THE "ELECTRIC FYDP" FORCE COSTING MODEL MATHEMATICS

The "Electric FYDP System" force costing model is an input-output costing model that is manipulated by varying one of two possible independent vectors. The costing model can be described by a set of four matrices and three vectors [3]. This set of matrices and vectors is used to model the production of m activities and the consumption of n consuming sectors.

Define the following terms:

PRIMARY INPUT - An input which is determined by factors outside the model but used in the production of intermediate commodities.

INTERMEDIATE COMMODITY - The output of a producing activity. Each producing activity uses primary inputs to produce an intermediate commodity; and, each producing activity uses intermediate commodities produced by other activities as inputs.

FINAL DEMAND - The demand of consuming sectors for primary inputs and intermediate commodities.

${}_{(m\times n)}A = (a_{ij})$ = Matrix of coefficients a_{ij} where a_{ij} is defined to be the amount of intermediate commodity i used per unit of final demand j.

$\underset{(m\times m)}{B} = (b_{ij})$ = Matrix of coefficients b_{ij} where b_{ij} is defined to be the amount of intermediate commodity i required per unit of intermediate commodity j produced.

$\underset{(k\times n)}{C} = (c_{ij})$ = Matrix of coefficients c_{ij} where c_{ij} is

defined to be the amount of primary input i required per unit of final demand j.

$\underset{(k\times m)}{D} = (d_{ij})$ = Matrix of coefficients d_{ij} where d_{ij} is defined to be the amount of primary input i required per unit of intermediate commodity j produced.

$\underset{(1\times n)}{\overline{Y}}$ = Vector of final demand.

$\underset{(m\times 1)}{\overline{X}}$ = Vector of intermediate commodities.

$\underset{(k\times 1)}{\overline{Z}}$ = Vector of primary inputs.

$\underset{(m\times 1)}{\overline{X}_F}$ = Vector of intermediate commodities used by final demand vector $\overline{Y}$.

$(m\times 1)\overline{X}_I$ = Vector of intermediate commodities used to produce $\overline{X}_F$.

$\underset{(k\times 1)}{\overline{Z}_F}$ = Vector of primary inputs used by final demand $\overline{Y}$.

$(k\times 1)\overline{Z}_I$ = Vector of primary input used to produce intermediate commodity vector $\overline{X}_I$.

Conceptually, these matrix and vector quantities are related in the manner depicted by Figure 8. Variables of the model are $\overline{Z}$, $\overline{Y}$, and $\overline{X}$. It will be shown that under certain conditions, the dependency of the model parameters as shown in Figure 7, can be reversed.

As depicted by Figure 8, $\overline{Y}$ is the independent vector. If the dependency were reversed $\overline{Z}$ would be independent and $\overline{Y}$ dependent. In either case, once the independent vector is specified, the remaining two vectors are dependent.

Pictorial Dependency of Variables Within the Model with $\overline{Z}$ Dependent

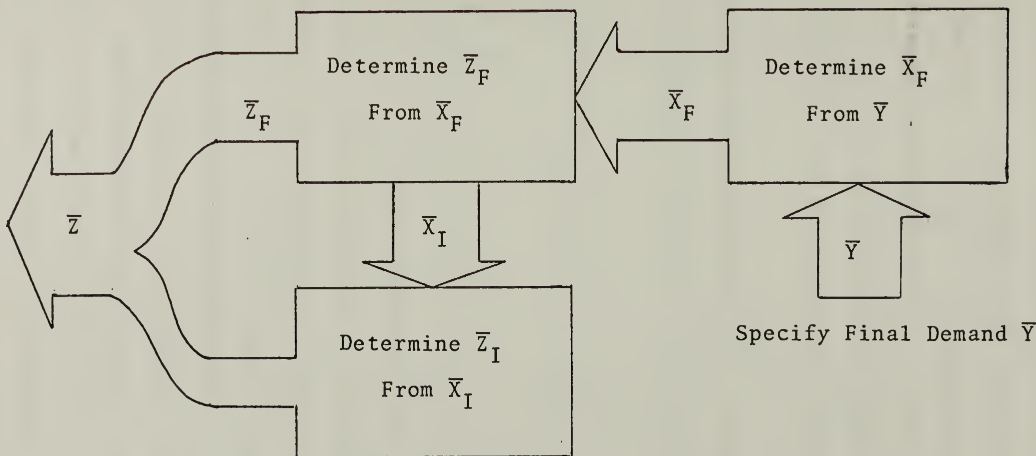


Figure 8

The model can estimate answers to any one of several questions. Specify Y as the independent vector and assign a value $\overline{Y}^*$. How much of each primary input denoted by the vector $\overline{Z}^*$, is used by final demand $\overline{Y}^*$? How much of each intermediate commodity, denoted by $\overline{X}^*$, must be produced to fulfill final demand $\overline{Y}^*$? What portions of $\overline{X}^*$ is used to produce itself? What portion of primary inputs $\overline{Z}^*$ is used directly to fulfill $\overline{Y}^*$, and what portion of $\overline{Z}^*$ is used to produce $\overline{Y}_I^*$?

All quantities related schematically in Figure 8 are mathematically dependent as follows: By definition of the A matrix

$$\overline{X}_F^* = A \cdot \overline{Y}^{*\prime} = [x_{F_1}^*, x_{F_2}^*, \ldots, x_{F_m}^*] \ , \qquad (1)$$

where

$$x_{F_i}^* = \sum_{j=1}^{n} a_{ij}\ y_j^* = \overline{a}_{i\cdot}\overline{Y}^{*\prime} \quad i=1,\ldots,m \ ,$$

and $\overline{a}_{i\cdot}$ is the i^{th} row of the A matrix and $\overline{Y}^{*\prime}$ is the transpose of $\overline{Y}^*$. By use of the B matrix and the intermediate vector $\overline{X}_F^*$ a Leontief input-output sub-model of the costing model can be constructed. A Leontief input-output model describes the inter-relationship of producing activities. It describes how the outputs of all producing activities are used as inputs. Each producing activity will require some of its own output and some of other produced outputs as inputs to its own production process. The outputs of all activities in excess of that required as inputs to production processes is left for consumption. In the Leontief input-output sub-model, the vector $\overline{X}^*$ is thought of as the vector

of "consumable" intermediate commodities part of which are "consumed" directly by the final demand vector $\overline{Y}*$. It is also thought of as inputs to the producing activities. Hence, $\overline{X}*$ represents the total vector of intermediate commodities that must be produced by all producing activities in order to meet the requirements of $\overline{Y}*$ and the requirements to produce $\overline{X}*$. This condition is expressed by

$$\overline{X}* = \overline{X}_I* + \overline{X}_F* \qquad (2)$$

where, by the definition of the B matrix

$$\overline{X}_I* = B \cdot X* \ . \qquad (3)$$

Substituting (3) into (2) results in

$$\overline{X}* = B \cdot \overline{X}* + \overline{X}_F* \qquad (4)$$

which can be algebraically rewritten as

$$(I-B)\overline{X}* = \overline{X}_F*$$

from which $\overline{X}*$ can be expressed formally as

$$\overline{X}* = (I-B)^{-1} \ \overline{X}_F* \ . \qquad (5)$$

Substituting (1) into (5) results in

$$\overline{X}* = (I-B)^{-1} \ A\overline{Y}*' . \qquad (6)$$

Note that total intermediate commodities $\overline{X}*$, intermediate commodities required only by $\overline{Y}*$, and intermediate commodities required to produce $\overline{X}*$ may be expressed in terms of the matrices defined and the specified final demand vecor $\overline{Y}*$. Similarly, by the definition of $\overline{Z}_F*$ and $\overline{Z}_I*$, $\overline{Z}*$ can be expressed as

$$\overline{Z}* = \overline{Z}_F* + \overline{Z}_I*. \qquad (7)$$

Also, by definition of the C and D matrix,

$$\overline{Z}_F{}^* = C\cdot\overline{Y}^{*\prime} \text{ and } \overline{Z}_I{}^* = D\cdot\overline{X}^* \tag{8}$$

Substituting equation (8) into equation (7) expresses $\overline{Z}^*$ as

$$\overline{Z}^* = C\cdot\overline{Y}^{*\prime} + D\cdot\overline{X}^* . \tag{9}$$

Substituting equation (6) into equation (9) for $\overline{X}^*$ results in

$$\overline{Z}^* = C\cdot\overline{Y}^{*\prime} + D\cdot(I-B)^{-1} A\cdot\overline{Y}^{*\prime} . \tag{10}$$

With equation (10) $\overline{Z}^*$, $\overline{Z}_F{}^*$, and $\overline{Z}_I{}^*$ can be determined in terms of the model parameters A, B, C, D, and the independent vector $\overline{Y}^*$.

Mathematically $\overline{Z}$ may be made the independent variable with X and Y dependent. Let $\overline{Z}^{**}$ denote a specified value of the independent vector $\overline{Z}$.

Equation (10) permits

$$\overline{Z}^{**} = C\cdot\overline{Y}' + D\cdot(I-B)^{-1}\cdot A\cdot\overline{Y}' ,$$

which can be expressed as

$$\overline{Z}^{**} = \{ C + D\cdot(I-B)^{-1}\cdot A \} \overline{Y}'. \tag{11}$$

In order to express $\overline{Y}$ in terms of the model parameters with $\overline{Z}^{**}$ as the independent vector, the inverse matrix $\{C+D\cdot(I-B)^{-1}\cdot A\}$ must exist. Sufficient conditions for its existence are that it be a square matrix and it be nonsingular. In general, $\{C+D\cdot(I-B)^{-1}\cdot A\}$ will not be a square matrix. Hence, expressing $\overline{Y}$ as

$$\overline{Y} = \{ C + D\cdot(I-B)^{-1}\cdot A\}^{-1}\cdot\overline{Z}^{**} \tag{12}$$

is dependent upon the existence of $\{C+D\cdot(I-B)^{-1}\cdot A\}^{-1}$. A generalized inverse of a non-square matrix exists. However, it is generally less tractable, and the necessary and sufficient conditions for its existence are more intricate [5]. Use of the generalized inverse in this model will not be considered in this paper.

IV. THE "ELECTRIC FYDP" FORCE COSTING METHODOLOGY

The "Electric FYDP" cost model can be interpreted in the economic terms of activity analysis. The model describes how primary inputs are used to produce intermediate commodities, and it describes how the intermediate commodities are consumed by the final demand sector and the producing section.

In matrix form the model can be written as two matrix equations, where quantities are as defined in Section III. The horizontal partitioning separates primary and intermediate commodities. The vertical partitioning separates the producing and consuming activities.

(1)

$$\underset{(m\times 1)}{\bar{X}} = \underset{[m\times(n+m)]}{(A \mid B)} \cdot \underset{[(n+m)\times 1]}{\left|\begin{array}{c}\bar{Y}' \\ \hline \bar{X}'\end{array}\right|}$$

and

(2)

$$\underset{(k\times 1)}{\bar{Z}} = \underset{[k\times(n+m)]}{(C \mid D)} \cdot \underset{[(n+m)\times 1]}{\left|\begin{array}{c}\bar{Y}' \\ \hline \bar{X}'\end{array}\right|}$$

which can be written as the matrix equation

(3)

$$\underset{[(k+m)\times 1]}{\left|\begin{array}{c}\bar{X} \\ \hline Z\end{array}\right|} = \underset{[(k+m)\times(n+m)]}{\left(\begin{array}{c|c}A & B \\ \hline C & D\end{array}\right)} \underset{[(n+m)\times 1]}{\left|\begin{array}{c}\bar{Y}' \\ \hline \bar{X}\end{array}\right|} .$$

The matrix

$$\left(\begin{array}{c|c}A & B \\ \hline C & D\end{array}\right)$$

is the technological matrix of the cost model. Each column of the technological matrix represents a production activity or a consuming activity. Each production activity uses primary inputs to produce intermediate commodities which in turn are used by all production processes to produce commodities for final demand consumption. The i^{th} element of the j^{th} column is called a technological coefficient of production. In the case of a production activity it represents the amount of the i^{th} input required to produce one unit of commodity j. In the case of a consumption activity, it represents the amount of the i^{th} input consumed per unit of final demand j.

It is more difficult to assess the indirect cost of a given force structure than it is to estimate the direct primary inputs and direct intermediate commodities and their associated costs. A direct primary input is a primary input such as untrained manpower or M60 tanks that are eventually used in an element of the force structure such as an Armored Division. An indirect primary input could be skilled labor that is used in production activities to train soldiers for use in the force structure. Another example of an indirect primary input is the use of an M60 tank as an instructional aid to train men in maintenance and operation procedures of M60 tanks. A direct intermediate commodity is any input of the force structure which is produced by the producing activities. For example, a trained artillery gunner assigned to an artillery battalion of the force structure would be a

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direct intermediate commodity. An indirect intermediate commodity could be a trained tank operator who trains tank operators for assignment to an element of the force structure. The principle distinction between a direct input and an indirect input is whether it is an input used to produce more intermediate commodities in a production activity or whether it is eventually used by an element of the force structure.

It is a difficult problem to estimate the indirect primary inputs and indirect intermediate commodities required for an element of the force structure, without an input-output model. These indirect requirements for the purpose of this paper comprise the "support tail." The model can readily estimate the primary inputs and intermediate commodity requirements of the "support tail" associated with a specified force structure. The model can also determine what proportion of the total indirect support is attributed to each element of the force structure. For example, if a force structure consists of bomber squadrons and aircraft carriers, the total support required for both can be determined. The proportion of total support that is required by bomber squadrons can also be determined with the "Electric FYDP" cost model.

As defined, the final output vector $\overline{Y}$ is a row vector with n elements. Let each of these n elements represent one element of a force structure vector. For example, in the case of the Army, the force structure vector represents the different categories of the Army Force Classification

System which is used for strategic and force planning purposes at the OSD, Joint Chiefs of Staff, and Army level [6]. If $y_i \varepsilon \overline{Y}$, then y_i could represent the number of Infantry Division Equivalents in an alternative force structure under investigation. A Division Equivalent is a unit of aggregate measurement used in the Army for force planning purposes. It could also represent the number of Division Equivalent Special Mission Forces or the number of Division Equivalent General Support Forces.

Each element of $\overline{X}$ represents an intermediate commodity that is used or "consumed" by an element of $\overline{Y}$. The j^{th} element of $\overline{X}$ might be base operating support, maintenance man-hours, administrative support, or training. An element of $\overline{X}_I$ might represent the amount of training required to train trainers; or, it may represent the number of maintenance man-hours spent maintaining training facilities. Each element of $\overline{Z}$ represents the primary inputs used to produce $\overline{X}_F$ and $\overline{X}_I$. $\overline{Z}_F$ is that portion of primary inputs used directly to produce $\overline{X}_F$ which in turn is consumed by $\overline{Y}$. Primary inputs are classified generally as manpower levels, equipment on hand, and dollars by appropriation category.

From the mathematical description presented in Section III, the defined matrices permit $\overline{Z}$ to be expressed as

$$\overline{Z} = \{C+D\cdot(I-B)^{-1}\cdot A\}\overline{Y}'.$$

C represents the direct primary input matrix and $D\cdot(I-B)^{-1}\cdot A$ represents the indirect primary input matrix. The primary inputs have been denoted in terms of dollars by appropriation category, real assets, and manpower levels. In order

to assess the total dollar cost of a given force structure, a vector of primary input cost coefficients must be defined with a cost factor for each type of primary input. Define:

$\underset{(1 \times k)}{\overline{k}}$ = vector of primary input cost per unit of primary input. (Note that all elements of k which are associated with dollar inputs will equal one.)

A vector of direct costs per unit of force structure is denoted by $\overline{\pi}$ and defined by

$$\overline{\pi} = \overline{k} \cdot C. \tag{13}$$

The elements of the vector $\overline{k}$ represents the cost, in terms of dollars, for each unit of primary input. Each element of the vector $\overline{\pi}$ represents the direct dollar cost per unit of the corresponding element of the force structure. The total direct <u>cost</u> of any force structure is computed by

$$C = \overline{\pi} \cdot \overline{Y}' \ .$$

The scalar C represents the total direct cost incurred by force structure $\overline{Y}$. The total direct cost incurred by the i^{th} element of the force structure $\overline{Y}$ is $(\pi_i) \cdot (y_i)$. The total indirect cost in computed by

$$Q_I = \overline{k} \cdot D \cdot (I-B)^{-1} \cdot A \cdot \overline{Y}'$$

where the scalar Q_I represents the total indirect cost of force strucutre $\overline{Y}'$.

Suppose we were able to define a vector of direct intermediate commodity values, $\overline{p}$, with one element for each element of $\overline{X}$. The i^{th} element of $\overline{p}$ would represent the economic

worth in terms of dollars of the i^{th} element of $\overline{X}$. With such a vector the value of $\overline{X}_F$ would be given by

$$Q = \overline{p} \cdot A \cdot \overline{Y}' = \overline{p} \cdot \overline{X}_F \ .$$

One of the objectives in a programming sense is to minimize total force cost while simultaneously maximizing the "value" in a programming sense of the force structure. It is reasonable that if a given amount of dollars is spent to procure inputs to a system which produces a vector of final outputs, the total amount spent will be the total economic worth or value of such a set of outputs. How is the total cost distributed among the elements of the final output vector? A method must be found which will permit the assertion that the k^{th} element of the final output vector is worth q dollars per unit of k.

There is such a way to distribute the total cost of primary inputs to the elements of the final output vector. In this sense a "value" or "worth" can be imputed to the elements of the output vector. It is "value" or "worth" in the sense that the imputed cost of the final output represents forgone economic opportunities. It also represents the trade-off value of each final output. The trade-off value of each element of the final demand vector is the total cost of all direct and indirect inputs attributable to each element of the final demand vector. Forgone economic opportunity of the j^{th} element of the force structure is the "value" the force structure would accrue if the j^{th} element were increased by one.

Mathematically, the objective becomes that of minimizing total force cost subject to a vector of direct input commodities. The variable is the vector of indirect input commodities. At the same time it is desired to maximize the economic value of the force structure. Each can be formulated as a linear programming problem and one is the dual of the other.

Let the imputed value parameters be denoted by the vector $\overline{p}$. It will be shown that $\overline{p}$ can be derived from the parameter of the basic model and the vector of primary input costs, $\overline{k}$. Total force cost is given by

$$Q = \overline{k} \cdot D \cdot \overline{X} + \overline{k} \cdot C \cdot \overline{Y}'$$

which is composed of the direct cost or constant term $\overline{k} \cdot C \cdot \overline{Y}'$ (because $\overline{k}$, C, and $\overline{Y}$ are known) and a variable indirect cost $\overline{k} \cdot D \cdot \overline{X}$. Formulate the following linear programming problem.

$$\min \ \overline{k} \cdot D \cdot \overline{X} + \overline{k} \cdot C \cdot \overline{Y}'$$

$$\text{subject to } (I-B)\overline{X} \geq A\overline{Y}'$$

$$\overline{X} \geq 0 \ .$$

Because the objective function is composed of a variable part and constant it may be written equivalently as

$$\min \ \overline{k} \cdot D \cdot \overline{X}$$

In words, the linear programming problem requires that a vector of intermediate commodities $\overline{X}$ can be found which is at least sufficient to provide the intermediate commodities required by the force structure $\overline{Y}$ and has minimum indirect cost. From duality theory of linear programming the dual to this primal problem can be formed as

$$\max \overline{p} \cdot A \cdot \overline{Y}'$$

$$\text{subject to } \overline{p}(I-B)' \leq \overline{K} \cdot D$$

$$\overline{p} \geq 0$$

The vector $\overline{p}$ is a vector of imputed costs of the direct intermediate commodities $\overline{X}_F = (I-B)\overline{X}$ which is used to produce force structure $\overline{Y}$. In words, the dual problem seeks a vector of costs, each element of which reflects the imputed "cost" or "worth" per unit of each direct intermediate commodity. This vector must maximize the imputed "cost" or "worth" of the force structure. Also, it must be such that the imputed cost of the direct intermediate commodities is less than or equal to the direct primary input costs.

Define a vector of imputed costs per unit of the elements of the vector $\overline{Y}$ as

$$\overline{W} = \overline{p} \cdot A.$$

The i^{th} element of $\overline{W}$ reflects the total imputed cost per unit of the i^{th} force structure element. The vector $\overline{W}$ is precisely the vector of imputed "values" for each element of a given force structure vector $\overline{Y}$.

From linear programming theory it is known that at optimality the optimal value of the dual will equal the optimal value of the primal if a solution to either exists. Hence,

$$\max \overline{p} \cdot A \cdot \overline{Y}' = \overline{p}^* \cdot A \cdot \overline{Y}' = C \cdot \overline{X}^* = \text{Min } C \cdot \overline{X}$$

where $\overline{p}^*$ and $\overline{X}^*$ are the optimal solutions to the dual and primal respectively. This condition implies that at optimality all costs of both direct and indirect primary inputs

will have been totally imputed to the direct intermediate commodities $\overline{X}_F$ in such a way that the "true" cost of an element of $\overline{Y}$ may be computed. With the vector $\overline{W}=\overline{p}\cdot A$, the cost of incrementing each element of $\overline{Y}$ can be readily assessed by visual inspection of the appropriate element of $\overline{W}$. $\overline{W}$ could possibly be used to evaluate the relative "value" of each element of a force structure. $\overline{W}$ could also be used to perform trade-off analysis between two alternative force structures. If $W_i < W_j$ and all other considerations except cost are equal, then force element j will contribute more per unit increase than force element i.

The mathematics of the "Electric FYDP" cost model are valid. Whether or not the structure of the model is appropriate for its intended use in the PPBS at DoD is also important. There are two methods for determining the validity of a mathematical model. One may use historical data to generate answers with the model. A favorable comparison of predicted results and observed results will validate the model if it is an appropriate model to use. This procedure may not be possible due to lack of data or for other reasons. An alternative validation procedure is to analyze the logic of the model. Examination of the implied and explicit assumptions of the model to determine if they are appropriate to a particular application can reveal whether or not the model is an appropriate one to use.

An analysis of the logic of input-output models in general and the "Electric FYDP" force costing model in particular will be presented in the following section. There

is no published evidence that such a validation has been previously performed. Predictive tests have not been formulated to perform validation by use of historical data, because the data and the exact specification of the model variables were not readily available. Hence, an analysis of the model's logic is presented.

V. ECONOMIC ANALYSIS OF "ELECTRIC FYDP" FORCE COSTING MODEL

In 1960, Michio Hatanaka, published a research document [7], which provides an excellent means of appraising the workability of any input-output model. Hatanaka's scheme of analysis will be used to appraise the workability of an input-output model to estimate force structure costs.

To evaluate the workability of a model as a means of description, one is concerned with evaluating the validity of the descriptive relationship and the method of estimating the parameters of the descriptive relationship. The validity of the parameters involves the logical structure of the model. It is related to the validity of the assumptions required by the logical structure of the model. This section will analyze the logical structure of the "Electric FYDP" force costing model. Problems of parameter estimation will be discussed in Section VI.

In the analysis which follows, it is assumed that the input-output model is being used to describe the technological processes of the system being modeled. Subsequently through use of the technological parameters of the system, it is assumed that predictions are to be derived. It is important to note that Hatanaka's analysis scheme is directed at the ability to accurately estimate the technology parameters which describe the relationships of the technologies of the I-O model in question. An input-output model may produce extremely accurate predictions, but it's model parameters

may be an inaccurate description of the technological processes being modeled.

It appears that the "Electric FYDP" cost model is being used in both a predictive and a descriptive role. The users of the model use its predictive ability to forecast the total cost of future force structures. On the other hand, current cost factors contained in service manuals are factors for force elements which have been developed empirically to estimate total force structure costs rapidly by service staff agencies. Cost factors are descriptive parameters which estimate the true input coefficient for a given input to a force structure element. The parameters of the force cost model used in a predictive role may not be comparable with cost factors which are descriptive parameters. This is a parameter estimation problem and will be discussed more fully in Section VI.

Hatanaka discusses five input-output model building characteristics that require analysis in order to insure that an input-output model is a workable model to use in modeling a given economic system. These model-building specifications are:

1. Unit of area.
2. Industrial classification.
3. The choice of endogenous industries.
4. Unit of time.
5. Period covered by the model.

Unit of area refers to the geographic boundaries of the economy within which each industry is aggregated. Industrial

classification refers to the method used to aggregate heterogeneous firms into a small group of industries. This is usually done by one of three possible bases:

1. Commodity basis.
2. Activity basis.
3. Establishment basis.

The separation of the total number of industries into two subsets of endogenous and exogenous industries is an important model building specification. Endogenous industries require constant coefficients of production in an I-O model; while exogenous industries do not require constant coefficients of production [7].

The unit of time in which input and output flow is measured is important because of the requirement for constant coefficients of production. Input-output coefficients measured in one unit of time may show constant coefficients of production, but when measured in another unit of time the coefficients may vary. The period covered by the model must include at least one more time period than the time period for which the input-output coefficients are measured, if the model is to be useful as a prediction model. Each of these specifications will be discussed in greater detail.

The economy to be modeled is a subsystem of the national economy, namely, the Department of Defense. The geographical boundary of DoD is worldwide. Each service and each agency of DoD is a sub-sub-system of the national economy. As a sub-economy of the nation, a military service can be thought of as consisting of many production activities and consuming

activities scattered worldwide. A production activity example is given by each of the major training centers such as Ft. Benning, Georgia or Ft. Sill, Oklahoma. A consuming activity example is given by any of the army divisions that are stationed worldwide, the Field Army Command Headquarters of Seventh Army in Europe, or United States Army Vietnam Command Headquarters in Saigon. As a producing activity, a major training center uses trained personnel, administrative support, and maintained equipment. Each producing activity plays a double role. It consumes primary inputs and produces intermediate commodities for consumption by consuming activities. Each of these consuming and producing activities may be viewed as a firm. In aggregate they may be viewed as an industry.

The idea of a production line is a set of coefficients which reflects the input requirements to produce one unit of a commodity. An example of a production line would be the inputs in terms of teaching-hours, floor space, instruction aids, etc., required to "produce" one trained artillery gunner. Define a basic activity as an activity which possesses one of a set of defined activity characteristics. Suppose that all production lines could be described by coefficients reflecting the amount of training, administration, and maintenance activities required to produce a unit of output. The set of basic activities would be training, administration, and maintenance.

The commodity basis of industrial classification uses a defined set of products. Each industry is defined as the

aggregate of all production lines for a given product. The establishment basis of classification uses a given set of products and associates each production line, joint or single, with one element of the set of products called the primary product. A joint production line is a production line by which the production of two or more products can be described with a single set of production coefficients. The activity basis of industrial classification defines an industry as the aggregate of all the production lines which posses a basic activity. The activity basis assumes that each production line consisting of more than one basic activity can be separated to the point that inputs and outputs for such a production line can be assigned to each basic activity. In the commodity basis of industry classification, it is assumed that if joint production is present, the production lines for each commodity can be identified and total inputs and outputs allocated to each line identified. The use of either the activity basis or the commodity basis of industrial classification depends upon the degree to which the inputs of an integrated production line can be split. Because of the size of the Army and the great number of army installations which are involved in joint activities, the establishment basis appears less feasible than the other two. Required for either of the other two methods is a crude classification of products or basic activities.

In the case of a military service economy a set of defined basic activities should not be difficult to determine.

The Army's Land Forces Classification System provides an excellent defined set of activity characteristics. Joint production is more likely to present a modeling problem if the commodity basis is used, because service production activities are organized to use one facility for as many uses as possible. For this reason, it may be extremely difficult to allocate the total use of a facility to the appropriate commodity production lines. It would however, be simplier and more likely to allocate total use of a facility to each of several activities. An example of this problem is the allocation of the total time a given lecture hall is used in a training center. The joint production lines using the lecture hall as an input must be separated. On a commodity basis the total time would have to be allocated to each type of trainee produced. On an activity basis the total time would have to be allocated to each type of activity such as training, entertainment, or maintenance. In the author's opinion, it is more likely that the activity basis allocation of total lecture hall time can be done more accurately than the commodity basis allocation.

A proper choice of a set of endogeneous industries should be made from the set of all industries comprising the sub-economy. Each "industry" in the military service can be classified as either producing or consuming. A producing industry is any industry that provides a commodity required by a consuming industry. The required commodity may be command and control, training, maintenance, or any other "product" required by the consuming industries. A "consuming" industry is any industry that requires or consumes a produced commodity. A consuming

industry could be a forward deployed field Army such as Seventh Army in Europe which uses trained personnel, maintenance-hours, command and control, administrative support, and many other produced commodities. Generally, those industries which consume produced commodities are the force structure elements; whereas, those industries which produce commodities for use in the force structure and by themselves are the "support-tail" industries. Hence, for the "Electric FYDP System" basic model the set of endogenous industries is that set of industries which use output from themselves as inputs to produce commodities required by the force structure. The set of exogenous industries is the complement to the endogenous set. The hypothesis of constant coefficients of production is applied only to the endogenous industry activity vectors. According to Hatanaka's analysis, the hypothesis of constant coefficients of consumption does not apply to the inputs used by the exogenous industries. The applicability of input-output analysis for modeling a military service hinges on whether or not a set of endogenous industries can be defined for which the coefficients of production are constant during the time unit chosen. In the case of the "Electric FYDP" force cost model it is not clear that this question has been addressed. Searches by the author for published or unpublished references have found no evidence that such a question has been addressed elsewhere. The intended set of endogenous industries is defined by the set of commodity producing activities denoted by the B and D matrices combined. However, the assertion of constant coefficients

should be evaluated as to its appropriateness in this case. It is not apparent that this has been done. The definition of an appropriate set of endogenous industries is a difficult validation problem and is an area for further research not addressed in this paper. This author will assume this is possible.

The unit of time during which a given set of input-output data is measured in order to derive the coefficients of production is a critical specification affecting the assertion of constant coefficients of production. The inputs during a given period can be considered as the inputs used for the production of outputs during the same period only if the time period is long enough. Input-output analysis requires that there exist some unit of time over which the input-output coefficients may be considered constant. It is not asserted that the input-output coefficients be constant for just any period of time. If the model is to be used to predict, this period must be at least one unit of time in addition to the base period or the period for which the input coefficients are computed. If the period covered by the model is found to be shorter than this minimum period then it is of little value in predicting. The constraining factor in lengthening the period of applicability of an input-output model is the capacity for gathering and processing the data necessary to derive the input coefficients.

In the case of the "Electric FYDP" cost model it is not evident that this time period has been explicitly defined. No references can be found to indicate that it has been done.

The time period necessary is two years. The base period is implicitly defined as one year. The model parameters are derived using base year data and are implicitly asserted to be constant for at least one additional year. Again, for the "Electric FYDP" cost model to be a workable model it should be tested to insure that the coefficients do in fact remain constant for the two year period. Empirical testing should be performed to confirm or deny the assertion of constant coefficients.

As pointed out by Hatanaka, any criticism of input-output analysis must be supported by empirical arguments. The danger of asserting the hypothesis of constant coefficients is great or small only to the extent that the objective of the input-output model is strongly or mildly affected by actual coefficient variations. In appraising the sources of possible weakness of input-output analysis, Hatanaka provides three viewpoints from which the appraisal should be considered. They are

1. Appraisal from the standpoint of theories of production.

2. Weaknesses from the viewpoint of model building.

3. The basic problem in input-output analysis.

Each will be examined in turn.

It should be pointed out that the input-output analysis force cost model used in the "Electric FYDP System" is a static open input-output model. It is static because its parameters represent flows for a fixed unit of time. It is open because the factors of production "purchased" by the endogenous

industries are not commodities "produced" by exogenous industries. Furthermore, the prices associated with the final demand commodity $\overline{Y}$ are not prices determined in the market place according to supply and demand. The prices associated with $\overline{Y}$ are the imputed values derived from the dual linear programming problem described earlier. There is no direct link between the "worth" or "value" of the final demand $\overline{Y}$ and the total cost of inputs to the endogenous industries.

From the viewpoint of theories of production, Hatanaka addresses possible weakness of input-output analysis related to the following:

1. Price substitution.
2. Level of output.
3. Factors of production outside the model.
4. Technological progress.
5. Natural and technological laws.

Industries may have alternative production vectors to produce the same commodity. The choice of which to use is dependent upon the prior knowledge of the relative prices of inputs. However, Hatanaka points out that I-O analysis has no way of identifying or nullifying the effects of changes in relative prices of inputs upon the input-coefficients of the endogenous industries. Relative prices of inputs become a static parameter like input coefficients. Any subsequent analysis or prediction must take the set of relative prices of primary inputs as given. If "zero draft" becomes a reality, the relative price of manpower with respect to other primary inputs will change. The degree and timing of such a change will not

be automatically detected by an I-O model. The effect will be an incorrect estimate of force cost.

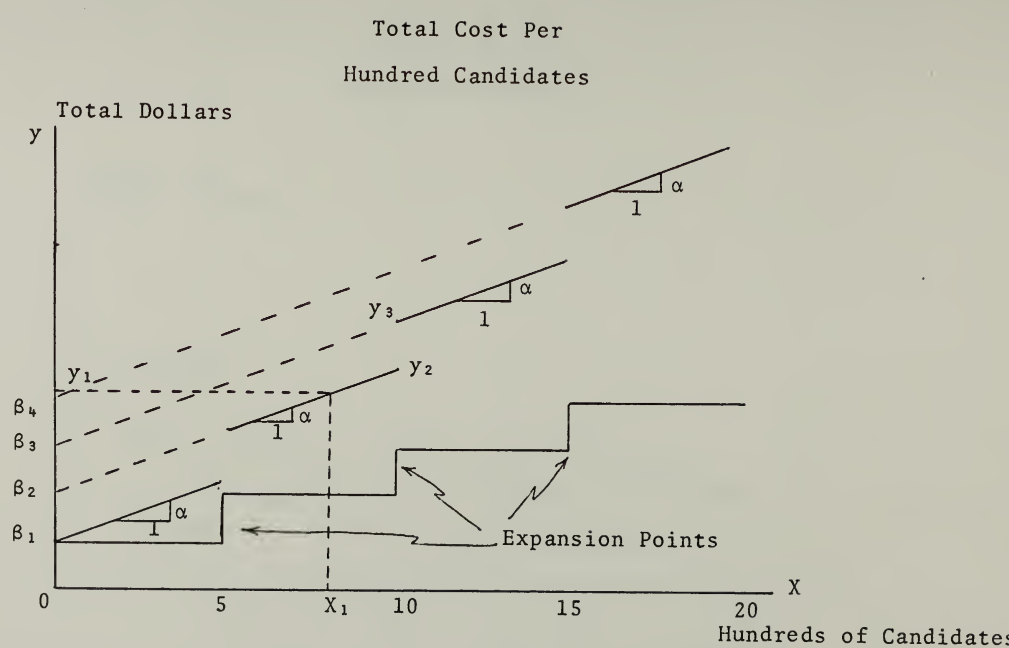
Constant input coefficients imply that the fixed coefficients of production are independent of the levels of output. According to Hatanaka [7] page 49, if the industry is to be treated as an endogenous industry, the input coefficients are determined by a linear function of the form $x_{ij} = \alpha_{ij}x_j + \beta_{ij}$. This linear form implies that a certain amount of input is required for production regardless of the level of output. In most cases, this is the real situation. However, the estimation of the parameters α_{ij} and β_{ij} is critical and no easy task. As Hatanaka points out, there is no workable method to make input coefficients completely independent of the level of output. Hence, to the degree that the endogenous industries of the economy being modeled by I-O analysis exhibit constant returns to scale within a reasonable interval about the base period level of output, the model will be valid or will produce bad estimates.

The "Electric FYDP" costing model estimates x_{ij} by $a_{ij}x_j$. This can be considered as a reduced form of the linear equation $x_{ij} = \alpha_{ij}x_j + \beta_{ij}$ where $a_{ij} = x^*_{ij}/x^*_j$ approximates the function $\alpha_{ij}x_j + \beta_{ij}$ within a small interval about x^*_j. The errors associated with this approximation could be great or small depending upon the value of β_{ij} and the deviation of x_j from x^*_j.

For a given technology there are upper bounds on certain input coefficients. When the upper bounds of an input coefficient is approached as the level of output increases one of

two possible courses must be taken in order to continue increasing output. Either the technology must change or the upper bound on inputs must increase. If technology is changed then the input coefficients will change. If the upper bound on inputs is increased then the approximation of the input coefficient by the method used in the "Electric FYDP" cost model may be a poor approximation.

As an example, consider the expansion of the Officer Candidate Program within the Army to meet the Vietnam buildup in 1965. At Ft. Benning, the increase was from one battalion of five companies to five battalions of from five to eight companies per battalion. A similar increase occurred at other training bases. This expansion occurred in increments. Each expansion incurred additional fixed costs which were attributed to the Officer Candidate Program. These fixed costs purchased expansion facilities for the program in the form of buildings, beds, desks, and other inputs. After each incremental expansion, an abrubt increase in manpower input capability occurred. This created a new upper bound on manpower input to the Officer Candidate Program which was gradually approached as more output from the program was required. The consequence of these incremental expansions in the Officer Candidate Program is illustrated in Figures 9 and 10. In this case the technology for producing candidates is kept constant. The input in question is the manpower input. In Figure 9 the oblique lines are all of the same slope, α, and represent the total cost function, $Y = \alpha x+\beta$. The parameter β, represents the fixed cost after each expansion. The term αx represents



Total Cost Per
Hundred Candidates
Total Dollars
y
y1
y2
y3
β4
β3
β2
β1
α
1
Expansion Points
0
5
X1
10
15
20
X
Hundreds of Candidates

Figure 9

Average Cost Per

Hundred Candidates

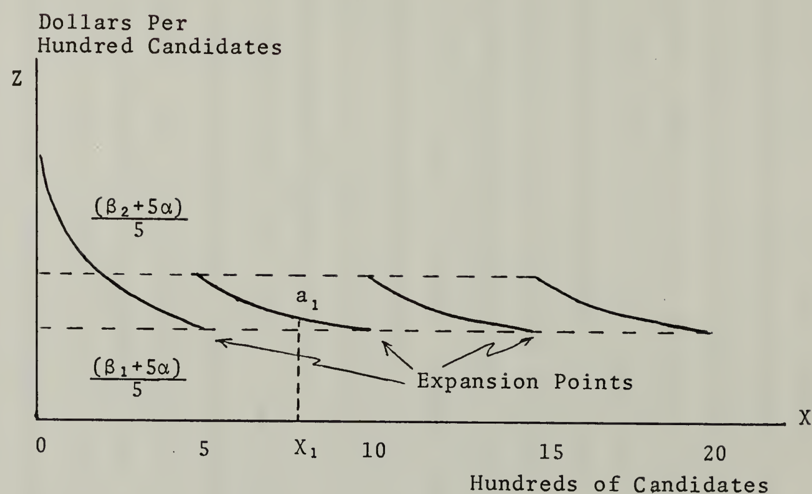


Figure 10

the variable cost associated with an input of level x. The parameter β increases on equal amount as a result of each expansion and is the reason for the step function appearance of β as x increases. The constant slope, α, is the variable portion of the technology and represents a constant cost increase per unit of output. This cost function is used because accounting procedures generally account for total cost in this manner. They are related to physically different input commodities. Fixed costs are related to construction of buildings, maintenance of buildings and facilities, and other functions which must be purchased in large segments in order to maintain a dormant capacity to supply these inputs as needed while varying output levels. Variable costs are related to the material and service inputs immediately acquired as a result of increasing output. This could be cost related to increased food consumption which increases as the number of manpower inputs increase.

Officer candidates are the output of the technology. The manpower technology coefficient for this technology is the average support input to produce one hundred candidates of output. This coefficient is estimated in the "Electric FYDP" cost model as $Y/X = Z$ where Z is the coefficient of production representing the number of support units of input to the technology required to produce 100 officer candidates. The average cost, assuming that all support inputs may be aggregated in terms of dollars, is the cost of producing each increment of one hundred candidates. In Figure 10 the average cost, Z, is plotted. It is a discontinuous function with

discontinuities at each expansion point. The average cost, assuming that $Y = x\alpha+\beta$ is a valid cost relationship, will vary between the two values of $(\beta_2+5\alpha)/5$ and $(\beta_1+5\alpha)/5$ depending upon the level of output required of the technology. These abrubt changes in average cost will not be detected by the estimate procedures for input coefficients used in the "Electric FYDP" cost model. Suppose the coefficient is estimated by using the cost of producing 800 candidates. From Figure 9 the quantities used are $y_1/x_1=a_1$. The estimated average cost to produce 100 candidates is a_1. Now suppose it is desired to estimate the cost of producing 1000 candidates using a_1 as the coefficient of production. The estimate of total cost to produce 1000 candidates would be $(a_1)1000=y_2$, when the true total cost is y_3. The effect has been to underestimate the cost of increasing candidate output from 800 to 1000.

Factors of production outside the model may cause changes in the input coefficients. This weakness is particularly serious when an open static input-output model is used. For instance, the accumulation and use of capital goods during an input period are not explicitly considered in the parameters of the model. This may cause changes in technologies which result from changes in the input coefficients of a producing activity if the time period covered by the model is too long. The decrease in capital assets through loss, wear, and theft can cause changes in the input coefficients for the same reason. For example, if the Army purchases new and technologically more effecient tanks for use in its armored

divisions and uses 30% less new tanks per armored division than the old tanks, the input of tanks per armored division would actually decrease. However, the model would not reflect this change in technology. The training requirements for the new tanks may be greater which could cause an increase in training-hours per trained tank crew. This change in technology would not be reflected in the parameters of the model. The model would produce an over-estimate of cost in the first case and an under-estimate in the second case.

The hypothesis of constant input coefficients is dependent upon the existence of invariable technological laws determining the amount of input required to produce a unit of output of a given product. If no technological law exists, or it is modified over time, the hypothesis of constant input coefficients is difficult to support. If the technological law describing the process being modeled is altered by beaurocratic decisions, coefficients change. In the case of endogenous industries of a military service it is not clear that a fixed "technological law" can be described which determines the amount of inputs required to produce a unit of output. An example is the civilian manpower input to the operation and maintenance of large training centers run by the Army such as Ft. Ord, California and Ft. Dix, New Jersey. The permanent civilian employee strength of such an installation is largely determined by Tables of Distribution. They are subject to annual review and change. Tables of Distribution assign a number of civilians per unit of type of activity. These annual changes are brought about by

compromise between two forces. One is the pressure to cut down on support costs. The second is the desire on the part of managers to keep their civilian employee strength at levels which will provide adequate support of their interests. During periods of strong economy pressure the annual compromise is normally in favor of cutting support costs. During periods of rising budget levels the direction of compromise is reversed. There exists no invariable technological law which reflects the permanent civilian manpower levels per unit of type of activity at such an installation for a period longer than one year.

From the standpoint of model-building, I-O analysis has possible weakness reflected by changes in input-coefficients due to aggregation of production lines. In the sub-economy to be modeled it is assumed that there exists an industrial classification such that the input coefficients can be considered constant. However, it may be that the classification for which this hypothesis is true is so refined that the I-O tables are intractable for even electronic computers. If this is the case, then industries must be aggregated into a smaller subset and that subset is used to model the same sub-economy. However, in this subset of aggregated industries, the hypothesis of constant input coefficients may not be true. Then the question becomes that of how gross can the aggregation of industries become before the variation in input coefficients becomes intolerable from a predictive viewpoint. Some aggregation methods will produce smaller variations in input coefficients from period to period than others. The

sensitivity of the model to aggregation errors should be tested in order to predict confidently with the model. In the case of the "Electric FYDP System" basic model, no evidence can be found by the author that aggregation errors have been examined. The support activities and consuming activities for the "Electric FYDP" force costing model, used to estimate costs for Army force structures, are very highly aggregated. The force structure is specified using the Land Forces Classification System. It is a very coarse aggregate of the Army force structure in terms of Division Force Equivalents and different stages of deployment. The degree to which this coarse aggregation causes costing errors is unknown.

An example of the coarseness of aggregation is the aggregation which occurs in specifying the inputs required by the support in a Division Force Equivalent. A Division Force Equivalent comprises the division and all supporting units which will be required by the division when it is employed in combat operations. Hence, the Division Force Equivalent is an aggregate measure of the division, the initial support required by the division after deployment (ISI), and the sustaining support increment (SSI). The ISI is an aggregation of all those initial combat support requirements of the division. The SSI is an aggregate of all additional combat support required by the division for an indefinite time after it has been committed to combat operations. Support includes many types of functions such as maintenance support. The maintenance function requires many different types of maintenance

activities. Aircraft maintenance, electronic maintenance, armament and fire control maintenance, wheeled vehicle maintenance, troop housing maintenance are but a few. In order to specify the maintenance support for a division force equivalent force structure vector, the aggregation of all maintenance activities must occur. It is the author's opinion that this aggregate is very coarse.

Hatanaka points out that the dilemma of input-output analysis is that each industry is analyzed in two respects [6]. One is from its role as a consuming industry and the other from its role as a producing industry. The input coefficient a_{ij} reflects both these aspects simultaneously. In the role as a consuming industry it appears as the i^{th} consumer. In its role as a producing industry, it appears as the j^{th} producer. The I-O dilemma appears in that the industrial classification for each role must be the same. Hence, if the coefficients of a_{ij} and $a_{i+1,j}$, for industry j as producer and for industries i and i+1 as consumer, tend to move in the opposite direction as changes in the industries take place (changes due to technological changes, introduction of new production lines, levels of output, etc.), their aggregation into industry I=i+(i+1) may produce a stable aggregate input coefficient $a_{I,j}=a_{ij}+a_{i+1,j}$. However, if they move in like directions as changes occur, their aggregation may produce an unstable $a_{I,j}$. These variations are very difficult to determine.

In the case of the "Electric FYDP" force costing model it is not evident that the workability of input-output analysis

has been addressed. From the subjective analysis of the basic model presented in this section, it is the author's opinion that the basic model should be subjected to intensive review if it is to be used as a descriptive model. Such a review should address the logic of the model from the standpoint of Hatanaka's scheme of analysis. It is not being asserted that the basic model is incorrect, but that the workability of the "Electric FYDP" force cost model for use as a descriptive force costing model is unknown. Until such a searching review is complete, answers derived from the "Electric FYDP System" should be used with some degree of reservation.

Because the "Electric FYDP" basic model depends so heavily upon the parameters or matrix elements, their estimation is critical. The next section of this paper will address some of the problems in estimating these parameters, and suggest methods for better parameter estimation.

VI. PARAMETER ESTIMATION PROBLEMS AND SENSITIVITY OF COST MODEL TO PARAMETER VARIATIONS

A. DISCUSSION

A requirement for the workability of input-output analysis for use as a predictive or descriptive model of an economic sector is constant coefficients of production for at least one time period longer than the base period. This requirement makes the estimation of input coefficients an important step in specifying the structure of an input-output model. Confidence in the general validity of the input-output model is related to the confidence one can express in the model parameters. The data used to estimate the parameters of the "Electric FYDP" cost model are subject to aggregation, reporting, and measurement errors. The consequences of these errors are unknown. Variations between two data points measured at different times can be attributed in part to changes in technologies which occur over time as described in Section V.

The "Electric FYDP System" cost model currently uses a simultaneous estimate of the model parameters. They are simultaneous at least in that the parameters for each row are completely estimated at one time. Entries for each element in each matrix are derived from the data of a single base year. For instance, suppose it was observed that in FY'68, force structure $\overline{Y}^*$ used $\overline{X}^*$ support commodities; and, $\overline{X}^*$ used $\overline{Z}^*$ of primary inputs. In addition, suppose that each element of all matrices has been identified using observed data. At

this stage, the model could be represented as

$$\begin{bmatrix} \bar{X}^* \\ \hline \bar{Z}^* \end{bmatrix} = \left[\begin{array}{c|c} \begin{matrix} x^*_{ij} \\ i=1,\ldots,m \\ j=1,\ldots,n \end{matrix} & \begin{matrix} x^*_{ij} \\ i=1,\ldots,m \\ j=n+1,\ldots,m+n \end{matrix} \\ \hline \begin{matrix} z^*_{ij} \\ i=m+1,\ldots,m+k \\ j=1,\ldots,n \end{matrix} & \begin{matrix} z^*_{ij} \\ i=m+1,\ldots,m+k \\ j=n+1,\ldots,n+m \end{matrix} \end{array}\right] \begin{bmatrix} \bar{Y}^* \\ \hline \bar{X}^* \end{bmatrix}$$

where an element in the upper right corner of the matrix represents the observed amount of commodity i required to produce commodity j. An element in the upper left corner of the matrix represents the observed amount of commodity i used by the j^{th} element of $\bar{Y}^*$. Similarly, an element of the lower left corner of the matrix represents the observed amount of primary input i used by the j^{th} element of $\bar{Y}^*$. An element in the lower right quadrant of the matrix represents the observed amount of primary resource i used to produce the j^{th} element of the support commodity vector $\bar{X}^*$.

The parameters of the model are estimated by normalizing each entry with appropriate elements from the known observed vectors, $\bar{Y}^*$, $\bar{X}^*$, and $\bar{Z}^*$. The input coefficient a_{ij} is estimated by

$$\hat{a}_{ij} = \frac{x^*_{ij}}{y_i} \qquad i \varepsilon \{1,2,\ldots,m\} \quad j \varepsilon \{1,2,\ldots,n\}$$

where "^" denotes that $\hat{a}_{ij}$ is an estimate of a_{ij}. Similar estimates are formed for each of the other parameters of the model as

$$\hat{b}_{ij} = \frac{x^*_{ij}}{x^*_i} \qquad i \,\varepsilon\, \{1,2,\ldots,m\}$$
$$j \,\varepsilon\, \{n+1,n+2,\ldots,n+m\}$$

$$\hat{c}_{ij} = \frac{Z^*_{ij}}{y^*_i} \qquad i \,\varepsilon\, \{m+1,m+2,\ldots,m+k\}$$
$$j \,\varepsilon\, \{1,2,\ldots,m\}$$

$$\hat{d}_{ij} = \frac{Z^*_{ij}}{x^*_i} \qquad i \,\varepsilon\, \{m+1,m+2,\ldots,m+k\}$$
$$j \,\varepsilon\, \{n+1,n+2,\ldots,n+m\}$$

where in all cases either $x^*_i \varepsilon \overline{X}^*$ or $Y^*_i \varepsilon \overline{Y}^*$.

Assuming that more than one data point is available, the estimation procedures which could be used may differ from the current parameter estimation method. With two sample points or more, a sample variance can be computed to estimate the variance associated with the estimated coefficient. With the sample variance, a confidence interval for the estimator of the coefficient can be derived. The benefit to be derived from the use of more than one sample point is increased confidence in the estimator.

In the case of the "Electric FYDP" cost model there are several different linear statistical models which could be used to estimate the parameters of the cost model. An excellent description of several different linear statistical models is given in Dhrymes [8] and Graybill [9]. In this author's opinion, all of these models may be separated into two groups. One group is that group of linear statistical models which estimates each technological coefficient as an individual random variable. Estimation procedures derived from these models provide estimates of each technology independent of all other technologies being modeled. Linear

statistical models from this group provide a descriptive estimator for each technology, hence these will be referred to as descriptive parameter estimation models. The functionally related model described by Graybill [9] and the errors in variables model with a stochastic error term described by Johnston [10] are two models in the group of descriptive parameter estimation models.

The second group of models are those which provide the predictive parameter estimation models. Hence, they will be referred to as predictive parameter estimation models. This group of linear statistical models is described by Dhrymes [8]. Models of this group provide simultaneous estimates of all model parameters, in this case, the entire I-O structure, by using simultaneous multiple equation regression analysis models. Ordinary least squares, two stage least squares, and Aitken estimators derived from simultaneous multiple equation regression analysis models are three such estimators discussed in Dhrymes [8]. These models estimate all technological coefficients of all technologies being modeled in a simultaneous manner. The linear statistical models of both groups associate an estimated variance with each input coefficient of all technologies being modeled.

As a demonstration of the difference which may exist between a predictive and a descriptive estimator consider the following example. Suppose a logical relationship is defined by

$$y = ax ,$$

where a is a coefficient which describes the causal relationship of x and y. Let (y_i, x_i), $i=1,\ldots,n$, be a random sample of observed values of y and x. Let $y_i/x_i=\hat{a}$ be a one sample point estimate of a. Then an estimate of a is given by

$$\hat{a} = \frac{\left(\sum_{i=1}^{n} \frac{y_i}{x_i}\right)}{n} .$$

Suppose n = 2 and

$$\{(y_i, x_i)\}=\{(1,1),(2,3)\} .$$

Then

$$\hat{a} = (3/3 + 2/3)/2 = 5/6 .$$

On the other hand, suppose one seeks a predictive parameter which predicts y using x. Then

$$y = \beta x + \mu$$

is the linear regression of y on x where μ is a random error term. Suppose it is known that μ is normally distributed with mean zero and variance σ^2. Let (y_i, x_i) be a random sample of observed values of y and x_i, $i=1,\ldots,n$. The observed samples each represent one equation of the form

$$y_i = \beta x_i + \mu_i \qquad i=1,\ldots,n$$

The model can be written compactly as

$$\overline{Y} = \beta\overline{x} + \overline{\mu}$$

where

$$\mathrm{cov}(\overline{\mu}) = E[\overline{\mu}'\ \overline{\mu}] = \sigma^2 I = \Sigma$$

a type of regression estimator called an Aitken estimator for β is given by

$$\hat{\beta} = (\overline{X}'\Sigma^{-1}\overline{X})^{-1}\overline{X}'\Sigma^{-1}\overline{Y} .$$

If one uses

$$\{(y_i, x_i)\} = \{(1,1), (2,3)\}$$

as a random sample and assume that $\sigma^2 = (.1)$ then,

$$\hat{\beta} = \left\{(1,3) \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}\right\}^{-1} \cdot$$

$$(1,3) \cdot \frac{1}{.01} \begin{pmatrix} .1 & 0 \\ 0 & .1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\hat{\beta} = \left(\frac{1}{100}\right) \cdot 70 = \frac{7}{10} ,$$

where by the formula for Cov $(\overline{\mu})$

$$\Sigma = \sigma^2 I = \begin{pmatrix} .1 & 0 \\ 0 & .1 \end{pmatrix}$$

$$\Sigma^{-1} = \frac{1}{.01} \begin{pmatrix} .1 & 0 \\ 0 & .1 \end{pmatrix} = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$

The descriptive estimate of a was $\hat{a} = 5/6$ while the predictive estimate β is $\hat{\beta} = .7$. Both were derived using the same data but with different estimation models.

B. DESCRIPTIVE PARAMETER ESTIMATION

Given that the model is to be used as a descriptive model, each model parameter should be considered as an individual

random variable. Descriptive parameters might be used to determine cost factors with the model which in turn might be compared with cost factors from service cost factor manuals [3]. Estimation of all model parameters then requires that an observed sample of $\overline{Y}$, $\overline{Z}$, and all matrix entries. The observed values of these entries are subject to random variations. These variations are attributable to the following observation errors. For economic data these variations are due to aggregation errors, reporting errors in measurement at the source of the data. Variations between data points measured at different points in time are in part due to changes in technologies which occur between the two points in time. The net effect of the aggregation of these errors is to produce random variations in the observed matrix entries.

If one is interested in estimating the descriptive parameters of a technology, then each coefficient should be considered as a random variable with some probability distribution which has a finite mean and finite variance. Each technology coefficient is then estimated by finding the best estimate of its mean. In the case of the "Electric FYDP" cost model one is interested in the technological coefficients a_{ij} defined by $a_{ij} = (x_{ij}/x_j)$. Each a_{ij} has a uni-variate distribution with a mean and variance. It is desirable that more than one sample point be used to estimate a_{ij}. If n observations of x_{ij} and x_j are available, then the best estimator of the mean is

$$\hat{a}_{ij} = \left(\sum_{k=1}^{n} \frac{x_{ijk}}{x_{jk}} \right) / n \quad ,$$

where "best" will be precisely defined late in this section. An alternative method of estimating the parameters, recommended if predictive parameters are desired, is to use regression in order to simultaneously estimate the coefficients. The use of regression derives estimates of all coefficients by a simultaneous equation regression model. All coefficients are estimated simultaneously using an observed sample of all matrix entries. If n observed samples of equal size were used to derive n estimates of all prediction coefficients, they would represent observed values of some multi-variate distribution. This multi-variate distribution requires intricate matrix algebra to derive estimators for its mean and covariance matrix. The techniques which are used to derive the best prediction estimator are discussed in general in the next section.

Currently, one sample is used to estimate each coefficient of each technology. Conceivably, similar estimating procedures would be used by DoD with sample sizes larger than one. By this author's classification, this would imply that descriptive parameters are desired. However, the model is chiefly being used to predict. In the author's opinion, prediction parameters should be estimated using regression techniques. These methods are much more intricate in a mathematical sense. They will be discussed in a later part of this section. If descriptive parameters are desired, then the current estimation derived using only one sample point is a poor estimator as will be shown below.

Suppose that X represents a random variable and $(x_1, x_2, \ldots, x_n)$ represents a random sample of observations of X. Further suppose that X has the normal probability distribution with unknown mean, μ, and unknown variance, σ^2. How can the observed sample be used to get a good estimate of the mean?

If two estimators of the mean of a random variable are proposed as the best estimate, one must use statistical properties of estimators to determine a comparative measure of "goodness." These properties are called bias, efficiency, and consistency.

Suppose that $\hat{\mu}$ is an estimator of the mean for the normal random variable X. Let

$$\{\hat{\mu}_i\}, \; i=1,\ldots,n,$$

represent a sequence of values for $\hat{\mu}$ computed from n repeated samples on X of equal size. The estimator $\hat{\mu}_i$ is also a random variable with probability density function related to the density function for X. Let $f_{\hat{\mu}}(s)$ represent the continuous density function for $\hat{\mu}$. Then by definition the estimator $\hat{\mu}$ is an unbiased estimator of μ if

$$E[\hat{\mu}] = \int_{-\infty}^{\infty} s f_{\hat{\mu}}(s)\,ds = \mu .$$

Let μ^* be a second estimator of μ. Suppose that

$$E[\mu^*] = \int_{-\infty}^{\infty} s f_{\mu^*}(s)\,ds = k\mu+\psi ,$$

where k and ψ are constants. In this case, μ^* would be a biased estimate of μ.

The second statistical property used to compare estimators is the efficiency property of estimators. Suppose that $\hat{\mu}$ and μ^* are both unbiased estimators of μ for the same sample and that the variances of $\hat{\mu}$ and μ^* are denoted by $\sigma^2_{\hat{\mu}}$ and $\sigma^2_{\mu^*}$ respectively. By definition, if $\sigma^2_{\hat{\mu}} < \sigma^{2*}_{\mu}$, then $\hat{\mu}$ is a more efficient estimator of μ than is μ^*.

The third statistical property used to judge the comparative "goodness" of estimators is the property of consistency. Let $\hat{\mu}_n$ be an estimator of the parameter μ based upon a sample of size n. Then by definition $\hat{\mu}_n$ is a consistent estimator of μ if

$$\lim_{n\to\infty} \{ \text{Probability } [|\hat{\mu}_n - \mu|<\varepsilon]\}= 1$$

for any $\varepsilon > 0$. Note that this is a statement about a sequence of probabilities as the sample size increases without bound. The consistency property of an estimator is an asympotic property in that it makes an assertion about the behavior of an estimator as the sample size becomes infinite. It says nothing about finite sample size estimates [11].

Estimators may be classified as linear or non-linear depending on whether or not the estimator is a linear or non-linear function of the sample. Let $\hat{\mu}$ be a linear estimator of μ. By definition $\hat{\mu}$ is the best linear unbiased estimator of μ if:

(1) $\hat{\mu}$ is a linear function of the sample,

(2) $E[\hat{\mu}] = \int_{\infty}^{\infty} s f_{\hat{\mu}}(s) ds = \mu$

(3) among the set of all estimators μ^* which also satisfy (1) and (2), $\sigma^2_{\hat{\mu}} \leq \sigma^2_{\mu^*}$. Let X be a random variable with

finite mean, μ, and finite variance, σ^2. From statistical theory if $(x_1, x_2, \ldots, x_n)$ is a random sample of X then the best linear unbiased estimator of μ is given by $\hat{\mu}$ where

$$\hat{\mu} = \left(\sum_{i=1}^{n} x_i \right) / n .$$

Note that the above is true regardless of the probability distribution of X. It can also be shown [11] that if

$$\lim_{n \to \infty} E[\hat{\mu}] = \mu$$

and

$$\lim_{n \to \infty} (\sigma^2_{\hat{\mu}}) = 0$$

then $\hat{\mu}$ is a consistent estimator of μ. Also, from statistical theory [11] it is true that for any random variable with finite mean and variance that

$$E[\hat{\mu}] = \mu$$

and

$$\sigma^2_{\hat{\mu}} = \frac{\sigma^2}{n} .$$

The degree of confidence one has in such an estimator for a given sample size is expressed by use of confidence intervals. The use of confidence intervals permits one to determine two pieces of information. First, one can determine a range of actual numerical values one feels the parameter may assume. Secondly, one can express one's confidence, on the basis of an observed sample, that the range of numerical values brackets the true unknown value of the parameter. Let X again be a normal random variable with unknown mean, μ,

and unknown variance, σ^2. Let $(x_1, x_2, \ldots, x_n)$ be a random sample of X. From statistical theory the 100(1-α)% confidence interval for μ is given in Larsen [11] (page 245) as

$$\text{Probability } [L_1 \leq \mu \leq L_2] = (1-\alpha)$$

where

$$L_1 = \hat{\mu} - t_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$L_2 = \hat{\mu} + t_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$S = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \hat{\mu})^2$$

$t_{1-\alpha/2}$ = 100(1-α) percentile point of the t distribution with n-1 degrees of freedom.

n = sample size.

$$\hat{\mu} = \left(\sum_{i=1}^{n} x_i \right) / n \ .$$

α = confidence coefficient such that $0<\alpha<<1$.

Note that the length of the confidence interval

$$| L_2 - L_1 |,$$

varies inversely as the sample size n. It also is dependent upon an estimate of the sample variance S, which requires more than one sample to compute.

This implies that for a given confidence coefficient, α, the length $|L_2 - L_1|$ decreases as n increases. Using a sample size of one to compute a 100(1-α)% confidence interval

on μ yields the largest possible confidence interval if α is specified.

It is the author's opinion that the normality assumption for each parameter could be applied to the parameters of the "Electric FYDP" model. It is a reasonable assumption to make and it is frequently assumed when the true distribution of a model parameter is unknown. If one accepts the normality assumption, then a one sample point estimate of the expected value of the parameter is the least confident estimate available. Of course, if only one sample is available, then a one sample point estimate is better than none at all; but, it is one in which the least confidence can be expressed. In the case of the "Electric FYDP" cost model it appears that a one sample estimate of each coefficient is used. Hence, a larger sample size should be used to estimate the parameters of the "Electric FYDP" cost model if at all possible.

In the author's opinion, it seems that a mathematical algorithm could be found to determine a estimator for total force cost variance implied by the use of the normality assumptions on each model parameter. Derivation of such a total force cost variance is an area for future research; however, conceptual ideas of how such a variance could be used in force cost analysis is presented later in this section.

C. ESTIMATION OF PREDICTIVE PARAMETERS

Given that it is desired to use the "Electric FYDP" model as a force cost predicting model, then it may be

possible to derive "good" predictive parameter estimates by use of multiple equation regression models to estimate predictive parameters for a system of simultaneous equations. Several types of multiple equation regression estimators for the parameters of a model such as the "Electric FYDP" cost model exist and are well explained in Dhrymes [8]. Depending upon the assumptions one is willing to make about the error vector of the statistical model, ordinary least squares or two stage least squares, among other more sophisticated methods, may be applied to determine linear unbiased, efficient, and consistent estimators. A very detailed discussion of the assumptions required in order to apply these estimation techniques is presented by Dhrymes. The use of such procedures should be carefully researched to insure that necessary statistical assumptions are valid. Such research is not presented in this paper.

Dhrymes provides a review of classical methods used to simultaneously estimate the parameters of a system of simultaneous equations by use of multiple equation regression models. The form of all models discussed by Dhrymes derive from the basic general linear model

$$y_t = \sum_{i=1}^{n} \beta_i x_{ti} + \mu_t \quad , \; t = 1,2,\ldots,T \; ,$$

where y is the explained variable, x_{ti} the explanatory variables which are usually assumed to be constant, t indexes the observations of the explained and explanatory variable, and β_i represents unknown predictive parameters which are to

be estimated. The μ_t represents an unobservable error term which is usually assumed to have the normal probability distribution with a mean of zero and a variance, σ^2. There is one such regression model for each equation of the system of simultaneous equations. Hence, there is a vector of error terms for each observed set of data points. Assumptions concerning the covariance matrix of the error vector indicate which of the estimators described by Dhrymes that should be applied. It is assumed that more than one set of observed independent data points are available.

Several problems may be encountered when applying regression techniques to a system of simultaneous equations. One such problem is called auto-correlation. This problem describes the condition in which the error terms for each successive sample in each equation are correlated in a statistical fashion with one another. The consequence of auto-correlation is that the derivation of ordinary least squares estimates will produce inefficient estimators. A second type of problem which can be encountered is called multi-collinearity. This problem results when some or all of the explanatory variables are highly correlated with one antoher. If this condition exists estimates derived by regression analysis may be biased, inefficient, and non-consistent. The general linear model assumes that the variance of error terms for each sample is constant. If this assumption is not true the condition known as heteroscedasticity is present. The consequences of heteroscedasticity

can also lead to biased, inefficient, and inconsistent estimators of the model parameters. There are other problems such as the ones described which can complicate and possibly render infeasible the use of regression analysis. An excellent detailed discussion of simultaneous equation estimation problems is presented in Johnston [10].

In this author's opinion, parameter estimation procedures for the "Electric FYDP" cost model could be derived by application of simultaneous multiple regression equation techniques. However, the possible presence and consequences of auto-correlation, multi-collinearity, heteroscedasticity, and other simultaneous equation regression estimation problems should be carefully researched. If such a regression method can be found which produces a good prediction estimator, further research might also permit total force cost variance to be associated with predicted force costs.

D. CONCEPTUAL USE OF TOTAL FORCE COST VARIANCE

In the author's opinion, the method selected to estimate the parameters of the model should, if possible, permit a variance to be associated with total force cost. The justification for this opinion and how such a variance would be useful is discussed below. No attempt is made to present a model. The ideas are conceptual only and represent, in the author's opinion, a fruitful area for further research into force cost analysis procedures.

The variance which can conceptually be associated with total force cost could be used to assist decision makers in

deciding which element of force structure to increment. In a deterministic model it is readily apparent which element of a force structure will cost more to increment.

For instance, suppose a military decision maker must decide which of two elements of the force structure to increase by one unit based upon cost considerations with all other considerations including effectiveness between the two systems being equal. With the basic model it is determined that force structures $\overline{Y}^{[1]}$ will cost $\overline{Z}^{[1]}$ in primary inputs. Then let it be desired to determine the incremental total cost of one more unit of the k^{th} element of $\overline{Y}^{[1]}$. Then, $\overline{Y}^{[2]}$ is equal to $\overline{Y}^{[1]}$ except the k^{th} element is increased by one. The basic model determines that $\overline{Y}^{[2]}$ will cost $\overline{Z}^{[2]}$ in primary inputs. A similar step is taken to investigate $\overline{Y}^{[3]}$ which equals $\overline{Y}^{[1]}$ except the r^{th} element is increased by one, and it is found that $\overline{Y}^{[3]}$ will cost $\overline{Z}^{[3]}$ in primary inputs. The decision maker can determine the total cost of each force structure by determining the total direct and indirect dollar cost of each primary resource. This is done by using the vectors $\overline{\pi}$ and $\overline{K}$ defined in Section III. The direct cost of each force structure $y^{[i]}$ would be

$$Q^{[i]}{}_D = \overline{\pi} \cdot \overline{Y}^{[i]}, \ i=1,2,3.$$

The indirect dollar cost of force structure $\overline{Y}^{[i]}$ is given by

$$Q^{[i]}{}_I = \overline{K} \cdot D \cdot (I-B)^{-1} \cdot A \cdot \overline{Y}'^{[i]}$$

and total cost of force structure $\overline{Y}^{[i]}$ is computed by

$$Q^{[i]} = Q^{[i]}{}_D + Q^{[i]}{}_I .$$

If the decision maker notes that $Q^{[2]} < Q^{[3]}$ he should properly conclude that more units of the k^{th} element can be bought for a given cost than the r^{th} element. But, assume that the variance of the estimated cost of $\overline{Y}^{[2]}$ is greater than the estimated cost of $\overline{Y}^{[3]}$. Let $\hat{Q}^{[i]}$ be the expected force cost for force $\overline{Y}^{[i]}$ and $\sigma^{[i]}$ be the standard deviation associated with $Q^{[i]}$. If it should occur that $\hat{Q}^{[2]} < \hat{Q}^{[3]}$ but $\sigma^{[2]} > \sigma^{[3]}$, which system should be incremented? In this situation a statistical decision rule could be derived which will establish the choice which should be made. The decision rule would be based upon the significance level of the statistical test specified by the decision maker. For instance, it may be specified that a system of the force structure will be incremented only if there is a 90% confidence coefficient that the estimate of it's cost is contained in a confidence interval that does not overlap the confidence interval associated with the cost of an alternative system. For instance, let the incremental cost of the i^{th} element of $\overline{Y}^{[1]}$ be $\hat{Q}^{[i]}-\hat{Q}^{[1]}=\Delta\hat{Q}^{[i]}$, where i=2,3. Then a statistical decision rule as described above would require that in order to increment system k of the force structure, it must be the case that $L_2 \leq K_1$ with

$$\text{Probability } [L_1 \leq \Delta Q^{[2]} \leq L_2] \geq .98$$

$$\text{Probability } [K_1 \leq \Delta Q^{[3]} \leq K_2] \geq .98,$$

where L_1 and L_2 are functions of $\hat{\sigma}^{[2]}$ and K_1 and K_2 are functions of $\hat{\sigma}^{[3]}$. The derivation of such a statistical test hinges upon the capability of associating a confidence interval

with each total force cost estimate as discussed at the end of Section VI-B.

E. SENSITIVITY OF THE COST MODEL TO PARAMETER ERRORS

This paper will infer the sensitivity of the "Electric FYDP" cost model to variations or errors in the estimation of its parameters from an examination of a small version of the model. At Appendix A the model has been represented by matrices of small size in order that changes in parameters can be easily made and their effect on solutions quickly calculated. It is desired to observe the percentage change in total force cost Q as small changes occur simultaneously and individually in the parameters of the model. In the first case the parameter b_{11} of the B matrix is varied by plus and minus 10%. The results were that a 10% increase in b_{11} caused a 3.76% increase in indirect cost Q_I and a 2.6% increase in total cost Q. In this instance a variation in one parameter is "damped" by the model. In the case where a parameter of the A matrix is varied, a_{22}, a 10% increase in a_{22} caused a .64% increase in Q and a 10% decrease in a_{22} caused a .3% decrease in Q. From these two sensitivity tests it appears that estimation of the parameters of the B matrix is the more critical.

Estimation errors are more likely to occur simultaneously in all parameters. If all parameters of the simplified model are simultaneously reduced 10% and increased 10% it is found that a 10% decrease in all parameters caused a 20.1% decrease in Q. In this case, it appears that estimation errors are

greatly attenuated in size. However, it is unlikely that all estimation errors will be in the same direction. Those parameter estimates which are too large and those which are too small would be on a random basis.

The inferences suggested by sensitivity analysis of the simplified basic model at Appendix A is by no means conclusive. Again, extensive research should be done to determine the sensitivity of the costing model as the order of the technological matrix increases in size, the level of operation of the force structure changes, and as the size and direction of the errors vary. It was noted that a 10% uniform increase in all parameters of the basic model produced a 24.7% increase in Q; however, an equal uniform decrease in all parameters produced a 20.1% decrease in Q. When percentage changes for direct costs Q_D and indirect costs Q_I were calculated it is observed that the 10% error in the parameters is transmitted unchanged into a 10% error in direct costs; however, the same 10% error is attenuated into a 33.3% increase in indirect costs when parameters are increased and a 26.5% decrease in indirect costs when all parameters are decreased. It appears that the indirect cost portion of the model is extremely sensitive to parameter variations.

An attempt was made by this author to derive an analytical model which would permit sensitivity analysis of the model in a more general manner. All efforts were thwarted by the necessity to find an expression for the general element of the $(I-B)^{-1}$ matrix in terms of the B matrix elements.

This author could not determine an analytically tactable expression of the general element of $(I-B)^{-1}$ as a function of the B matrix elements.

The scenario of the sensitivity analysis at Appendix A was selected by the author. The 10% error used in all cases was arbitrarily selected. Because it seemed reasonable that variations in the B matrix elements would telescope into a much greater variation in total force cost, the sensitivity of total and indirect force cost to variation in a single element of the B matrix was calculated. This seemed reasonable because the computations of $(I-B)^{-1}$ requires numerous multiplication operations of the B matrix elements. In a low order matrix this computation can be done by hand. For matrices of order higher than four, the computation of $(I-B)^{-1}$ becomes increasingly more time consuming. To show the contrast, sensitivity of the model to variations in a single parameter of the A matrix was performed. In order to illustrate the extreme effects of parameter variations, the two extreme cases in which all parameters take on extreme 10% plus and 10% minus values is computed. It is the author's opinion that this scenario is sufficient to support a preliminary conclusion about the sensitivity of the "Electric FYDP" cost model to variations in its parameters.

VII. SUMMARY

A. CONCLUSIONS

The purpose of this paper has been to appraise the "Electric FYDP System" force costing model as it is currently being used at DoD and propose areas for further research to improve upon it. The "Electric FYDP System" is certainly a step in the right direction for providing a clear understandable dialoque between OSD(SA) and the military agencies during the annual budgeting cycle. It is also important that the "Electric FYDP System" force costing model be a "good" model in the sense that confidence can be associated with its estimates of future defense force structure costs.

"The Electric FYDP" basic model should be thoroughly analyzed from the standpoint of its workability in modeling a military service economy. Preliminary analysis has been performed by the author using empirical knowledge of management methods and technology of one service. In the author's opinion, this preliminary analysis indicates that the confidence which can be associated with the force cost estimates produced by the "Electric FYDP" cost model is not known. This paper points the way for analysis of the cost model which may permit derivation of confidence statements. Without the actual "Electric FYDP" cost model and necessary data, the analysis proposed cannot be performed. Special collection efforts may be required to obtain such data.

When the model is to be used in a descriptive sense, the problem of descriptive parameter estimation must be solved. The current method of parameter estimation seems to consider each parameter as a random variable to be individually estimated. Currently, one sample is used to estimate the coefficients of production parameters. This is a serious criticism of the estimation procedure currently being used in that no measure of variance can be associated with a one sample point estimate.

When the model is used as a predictive model, it is the author's opinion that simultaneous multiple equation regression analysis should be investigated as a possible method of estimating predictive model parameters. However, careful research should accompany the development of such a regression model to insure that the presence of problems associated with simultaneous multiple equation regression analysis as pointed out briefly in Section VI is considered.

The consequences of errors in parameters of the "Electric FYDP" cost model have been indicated by a preliminary sensitivity analysis of a simplified version of the model. In the author's opinion, this preliminary analysis warrants further intensive investigation. The errors associated with the answers produced by the model could conceptually be critical to the problem of force structure modernization scheduling. The benefit of associating a variance with total force cost is the root of any statistical decision rule for deciding which of two alternative systems of the force structure to increment when all other considerations except cost

are equal. It is the author's opinion that research should be conducted for the derivation of such a statistical decision rule for use by DoD decision makers.

All of the above implications are related to the assertion that to use a force costing model that accounts for possible variations in its parameter is better than using a deterministic force costing model. It is the opinion of the author that, 1) the "Electric FYDP System" basic model can be converted from a deterministic model to a model which can permit variations in parameters to be accounted for in its costing methodology, and 2) the further benefits that could be derived in the area of DoD fiscal management and scheduling of force structure modernization warrant the research effort which would be required.

B. AREAS OF FURTHER RESEARCH

Several areas for further research and study have been suggested by this paper. A need exists for a force modernization scheduling algorithm as suggested and described in Section II. Currently, there is no efficient rapid method to create and examine all possible alternative schedules. The problem has been described and a possible criterion for alternative selection has been suggested, but the algorithm must be produced.

The questions raised by Section V, about the workability of I-O analysis, should be carefully researched in order to determine conclusively whether or not the assumptions required by the basic model are in fact met. If the assumption

of constant coefficients of production is not valid, the effect of variations in the coefficients of the model should be determined. In the area of parameter estimation two subjects should be researched. If the model is to be used as a descriptive model, then proper methods to estimate good descriptive parameters must be researched and formulated. Secondly, if the model is to be used as a prediction model, then simultaneous multiple equation regression analysis techniques should be found to provide good estimators of predictive parameters.

An additional area for further research is a complete sensitivity analysis of the model. This should include an examination of how errors in the support matrix increase or decrease the total force cost and indirect force cost. This analysis should also be done for models with matrices of increasing sizes. Initial investigation indicates that this could be a serious source of errors in the model.

APPENDIX A: SENSITIVITY ANALYSIS OF SIMPLIFIED VERSION OF BASIC MODEL

A. SENSITIVITY OF INDIRECT COSTS AND TOTAL COST TO SINGLE B MATRIX PARAMETER ERROR

Let the following matrices represent a simplified version of the "Electric FYDP" cost model.

$$B = \begin{pmatrix} .30 & .10 \\ .40 & .20 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 4 & 3 & 6 \\ 8 & 3 & 7 \\ 5 & 3 & 2 \end{pmatrix} \qquad D = \begin{pmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{pmatrix}$$

$$\overline{Y} = (10,\ 4,\ 5)$$

Allow b_{11} of the B matrix to vary plus and minus 10% and determine the percentage change in total force cost and the percentage change in indirect cost. For convenience define the vector of primary input prices as $\overline{p} = (1,1,1)$. Denote the B matrix with b_{11} increased by 10% as B* and with b_{11} decreased by 10% as B**. From the operation of the model, total commodities required are computed by

$$\overline{X} = (I-B)^{-1}\cdot A\cdot \overline{Y}$$

Hence,

$$\overline{X} = (I-B)^{-1}\cdot A\cdot Y$$

$$\overline{X} = 1/.52 \begin{pmatrix} .8 & .1 \\ .4 & .7 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 53.7 \\ 55.6 \end{pmatrix}$$

and

$$\overline{X}^* = (I-B^*)^{-1} \cdot A \cdot \overline{Y}$$

$$\overline{X}^* = 1/.496 \begin{pmatrix} .80 & .10 \\ .40 & .67 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 56.2 \\ 56.9 \end{pmatrix}$$

and

$$\overline{X}^{**} = (I-B^{**})^{-1} \cdot A \cdot \overline{Y}$$

$$\overline{X}^{**} = 1/.544 \begin{pmatrix} .10 & .10 \\ .40 & .73 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 51.3 \\ 54.4 \end{pmatrix}$$

With the required commodities vector for each case computed, the primary input vector $\overline{Z}$ and the indirect primary input vector $\overline{Z}_I$ can be computed by

$$\overline{Z} = C \cdot \overline{Y} + D \cdot \overline{X}$$

and

$$\overline{Z}_I = D \cdot \overline{X}.$$

Hence,

$$\overline{Z} = C \cdot \overline{Y} + D \cdot \overline{X}$$

$$\overline{Z} = \begin{pmatrix} 4 & 3 & 6 \\ 8 & 3 & 7 \\ 5 & 3 & 2 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{pmatrix} \begin{pmatrix} 53.7 \\ \\ 55.6 \end{pmatrix}$$

$$\overline{Z} = \begin{pmatrix} 245.0 \\ 265.1 \\ 255.3 \end{pmatrix}$$

and

$$\overline{Z}_I = \begin{pmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{pmatrix} \begin{pmatrix} 53.7 \\ 55.6 \end{pmatrix} = \begin{pmatrix} 163.0 \\ 138.1 \\ 183.3 \end{pmatrix}$$

For B*,

$$\overline{Z}^* = \begin{pmatrix} 4 & 3 & 6 \\ 8 & 3 & 7 \\ 5 & 3 & 2 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{pmatrix} \begin{pmatrix} 56.2 \\ 56.9 \\ \end{pmatrix}$$

$$\overline{Z}^* = \begin{pmatrix} 251.3 \\ 268.9 \\ 263.4 \end{pmatrix}$$

$$\overline{Z}_I^* = \begin{pmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{pmatrix} \begin{pmatrix} 56.2 \\ 56.9 \\ \end{pmatrix} = \begin{pmatrix} 169.3 \\ 141.9 \\ 191.4 \end{pmatrix}$$

Similar calculations for B** yield

$$\overline{Z}^{**} = \begin{pmatrix} 238.4 \\ 261.4 \\ 247.7 \end{pmatrix}$$

and

$$\overline{Z}_I^{**} = \begin{pmatrix} 156.4 \\ 134.4 \\ 175.7 \end{pmatrix}$$

Using the defined vector of primary input prices the total and indirect cost for each of the three cases is computed by

$$\text{Total Cost} = Q = \overline{p} \cdot \overline{Z}$$

$$\text{Total Indirect Cost} = Q_I = \overline{p} \cdot \overline{Z}_I$$

Hence,

$$Q = \overline{p} \cdot \overline{Z} = (1,1,1) \begin{pmatrix} 245.0 \\ 265.1 \\ 255.3 \end{pmatrix} = 765.4$$

$$Q_I = \overline{p} \cdot \overline{Z}_I = 502.6$$

and

$$Q^* = \overline{p} \cdot \overline{Z}^* = (1,1,1) \begin{pmatrix} 251.3 \\ 268.9 \\ 263.4 \end{pmatrix} = 783.6$$

$$Q_I^* = \overline{p} \cdot \overline{Z}^*_I = 502.6$$

and

$$Q^{**} = \overline{p} \cdot \overline{Z}^{**} = (1,1,1) \begin{pmatrix} 238.4 \\ 261.4 \\ 247.4 \end{pmatrix} = 747.5$$

$$Q_I^{**} = \overline{p} \cdot \overline{Z}_I^{**} = 466.5$$

In summary, for a 10% increase in b_{11} total cost, Q, increase by

$$\frac{Q^* - Q}{Q} \times 100 = \frac{783.6 - 765.4}{765.4} \times 100 = 2.6\%$$

and indirect cost, Q_I, increase by

$$\frac{Q^* - Q}{Q_I} \times 100 = \frac{502.6 - 484.4}{484.4} \times 100 = 3.76\% .$$

For a 10% decrease in b_{11}, total cost, Q, decreases by

$$\frac{747.5 - 765.4}{765.4} \times 100 = -2.1\%$$

and indirect cost, Q_I, decreases by

$$\frac{466.5 - 484.4}{484.4} \times 100 = -3.7\%$$

B. SENSITIVITY OF INDIRECT COSTS AND TOTAL COSTS TO ERRORS OF A SINGLE A MATRIX PARAMETER

Permit a_{22} to vary plus and minus 10% and calculate the percentage changes in direct costs and total cost of

a specified force structure. In this case A* will have a_{22} increased by 10% above its value in A and A** will have a_{22} decreased by 10% below its value in A. All other model parameters will remain unchanged.

The first calculation is to determine the required commodities vector for each case by

$$\overline{X} = (I-B)^{-1} \cdot A \cdot \overline{Y}$$

Hence, substituting the appropriate values in the above matrix equation will give

$$\overline{X} = 1/.52 \begin{pmatrix} .8 & .1 \\ .4 & .7 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 53.6 \\ 55.4 \end{pmatrix}$$

Similarly for $\overline{X}*$ and $\overline{X}**$,

$$\overline{X}* = 1/.52 \begin{pmatrix} .8 & .1 \\ .4 & .7 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 1 & 2.2 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 53.6 \\ 56.6 \end{pmatrix}$$

and

$$\overline{X}** = 1/.52 \begin{pmatrix} .8 & .1 \\ .4 & .7 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 1 & 1.8 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 53.6 \\ 55.4 \end{pmatrix}$$

The required primary input vector is then calculated by

$$\overline{Z} = C \cdot \overline{Y} + D \cdot \overline{X}$$

and the required indirect primary inputs are computed by

$$\overline{Z}_I = D \cdot \overline{X}$$

Again, substituting the appropriate quantities for each case into the above matrix equations yields

$$\bar{Z} = \begin{bmatrix} 4 & 3 & 6 \\ 8 & 3 & 7 \\ 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{bmatrix} \begin{bmatrix} 53.6 \\ 55.4 \\ \end{bmatrix}$$

$$\bar{Z} = \begin{bmatrix} 82 \\ 127 \\ 72 \end{bmatrix} + \begin{bmatrix} 163.0 \\ 138.1 \\ 183.4 \end{bmatrix} = \begin{bmatrix} 245.0 \\ 265.1 \\ 255.3 \end{bmatrix}$$

$$\bar{Z}^* = \begin{bmatrix} 4 & 3 & 6 \\ 8 & 3 & 7 \\ 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{bmatrix} \begin{bmatrix} 53.6 \\ 56.6 \\ \end{bmatrix}$$

$$\bar{Z}^* = \begin{bmatrix} 52 \\ 127 \\ 72 \end{bmatrix} + \begin{bmatrix} 163.8 \\ 140.0 \\ 183.4 \end{bmatrix} = \begin{bmatrix} 245.8 \\ 267.0 \\ 255.4 \end{bmatrix}$$

$$\bar{Z}^{**} = \begin{bmatrix} 4 & 3 & 6 \\ 8 & 3 & 7 \\ 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{bmatrix} \begin{bmatrix} 53.6 \\ \\ 54.5 \end{bmatrix}$$

$$\bar{Z}^{**} = \begin{bmatrix} 82 \\ 127 \\ 72 \end{bmatrix} + \begin{bmatrix} 161.7 \\ 135.8 \\ 182.6 \end{bmatrix} = \begin{bmatrix} 243.7 \\ 262.8 \\ 254.6 \end{bmatrix}$$

The required indirect primary inputs $\tilde{Z}_I$ for each case are then computed by substituting the appropriate quantities into $\bar{Z}_I = D \cdot \bar{X}$. Hence,

$$\bar{Z}_I = \begin{bmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{bmatrix} \begin{bmatrix} 53.6 \\ 55.4 \\ \end{bmatrix} = \begin{bmatrix} 163.0 \\ 138.1 \\ 183.4 \end{bmatrix}$$

$$Z^*_I = \begin{bmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{bmatrix} \begin{bmatrix} 53.6 \\ 56.6 \\ \end{bmatrix} = \begin{bmatrix} 163.8 \\ 140.0 \\ 183.4 \end{bmatrix}$$

$$Z^{**}_I = \begin{pmatrix} 2 & 1 \\ .5 & 2 \\ 3 & .4 \end{pmatrix} \begin{pmatrix} 53.6 \\ 54.5 \end{pmatrix} = \begin{pmatrix} 161.7 \\ 135.8 \\ 182.6 \end{pmatrix}$$

Using the price of primary inputs as $\overline{p} = (1,1,1)$ the total force cost and indirect cost for each case is calculated by

$$\text{Total Cost} = Q = \overline{p} \cdot \overline{Z}$$

$$\text{Indirect Cost} = Q_I = \overline{p} \cdot \overline{Z}_I$$

Hence,

$$Q = (1,1,1) \begin{pmatrix} 245.0 \\ 265.1 \\ 255.3 \end{pmatrix} = 765.4$$

$$Q_I = (1,1,1) \begin{pmatrix} 163.0 \\ 138.1 \\ 183.4 \end{pmatrix} = 484.4$$

$$Q^* = (1,1,1) \begin{pmatrix} 245.8 \\ 267.0 \\ 255.4 \end{pmatrix} = 768.2$$

$$Q^*_I = (1,1,1) \begin{pmatrix} 163.8 \\ 140.0 \\ 183.4 \end{pmatrix} = 487.2$$

$$Q^{**} = (1,1,1) \begin{pmatrix} 243.7 \\ 262.8 \\ 254.6 \end{pmatrix} = 761.2$$

$$Q^{**}_I = (1,1,1) \begin{pmatrix} 161.7 \\ 135.8 \\ 182.6 \end{pmatrix} = 480.1$$

The percentage changes in Q and Q_I as a result of the 10% increase and decrease in a_{22} is found by substituting the appropriate quantity for each case into

$$\text{Percent Change in } Q = \Delta Q/Q \times 100$$

$$\text{Percent Change in } Q_I = \Delta Q_I/Q_I \times 100$$

Doing this for each case yields:

CASE I. a_{22} increased by 10%

$$\text{Percent Change in } Q = \frac{768.2 - 765.4}{765.4} \times 100 = .63\%$$

$$\text{Percent Change in } Q_I = \frac{487.2 - 484.4}{484.4} \times 100 = .58\%$$

CASE II. a_{22} decreased by 10%

$$\text{Percent Change in } Q = \frac{761.1 - 765.4}{765.4} \times 100 = .30\%$$

$$\text{Percent Change in } Q_I = \frac{480.1 - 484.4}{484.4} \times 100 = .88\%$$

C. SENSITIVITY OF TOTAL COST TO UNIFORM SIMULTANEOUS CHANGES IN ALL MODEL PARAMETERS

In this instance it is desired to observe the percentage change in total cost Q when all parameters of the model are simultaneously increased by 10% and simultaneously decreased by 10%. The superscript "*" will denote that all parameters of the current calculation have been increased by 10% and the superscript "**" will denote a decrease of 10% in all parameters.

Again, the first step is to calculate the required commodities vector $\overline{X}$ by substituting the appropriate quantities into

$$\overline{X} = (I-B)^{-1} \cdot A \cdot \overline{Y}$$

Since base case will remain unchanged it will not be re-calculated. From the previous section

$$\overline{X} = \begin{pmatrix} 53.6 \\ 55.4 \end{pmatrix}$$

In the other two cases

$$\overline{X}^* = (I-B^*)^{-1} \cdot A^* \cdot \overline{Y}$$

$$\overline{X}^* = 1/.47 \begin{pmatrix} .78 & .11 \\ .44 & .67 \end{pmatrix} \begin{pmatrix} 1.1 & 3.3 & 2.2 \\ 1.1 & 2.2 & 1.1 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix}$$

$$\overline{X}^* = \begin{pmatrix} 64.3 \\ 69.0 \end{pmatrix}$$

and

$$\overline{X}^{**} = (I-B^{**})^{-1} \cdot A^{**} \cdot \overline{Y}$$

$$\overline{X}^{**} = 1/.58 \begin{pmatrix} .82 & .09 \\ .36 & .73 \end{pmatrix} \begin{pmatrix} .9 & 2.7 & 1.8 \\ .9 & 1.8 & .9 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix}$$

$$\overline{X}^{**} = \begin{pmatrix} 44.7 \\ 44.7 \end{pmatrix}$$

The next step is to calculate the required primary input vector $\overline{Z}$ for each case. Again, because $\overline{Z}$ will remain unchanged for the base case of the previous section it will

not be re-computed. Hence, substituting the appropriate quantities into

$$\overline{Z} = C\cdot\overline{Y} + D\cdot\overline{X}$$

will yield

$$\overline{Z} = \begin{pmatrix} 245.0 \\ 265.1 \\ 255.3 \end{pmatrix}$$

$$\overline{Z}* = C*\cdot\overline{Y} + D*\cdot\overline{X}*$$

$$\overline{Z}* = \begin{pmatrix} 4.4 & 3.3 & 6.6 \\ 8.8 & 3.3 & 7.7 \\ 5.5 & 3.3 & 2.2 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 2.2 & 1.1 \\ .55 & 2.2 \\ 3.3 & .44 \end{pmatrix} \begin{pmatrix} 64.3 \\ 69.0 \\ \end{pmatrix} = \begin{pmatrix} 305.5 \\ 326.9 \\ 322.0 \end{pmatrix}$$

$$Z{*}{*} = C{*}{*}\cdot\overline{Y} + D{*}{*}\cdot\overline{X}{*}{*}$$

$$Z{*}{*} = \begin{pmatrix} 3.6 & 2.7 & 5.4 \\ 7.2 & 2.7 & 6.3 \\ 4.5 & 2.7 & 1.8 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 1.8 & .9 \\ .45 & 1.8 \\ 2.7 & .36 \end{pmatrix} \begin{pmatrix} 44.7 \\ 44.7 \\ \end{pmatrix} = \begin{pmatrix} 194.5 \\ 214.9 \\ 201.6 \end{pmatrix}$$

Using the primary input price vector $\overline{p} = (1,1,1)$ to calculate total cost from

$$\text{Total Cost} = Q = \overline{p}\cdot\overline{Z}$$

and substituting appropriate quantities for each case yields

$$Q = \overline{p}\cdot\overline{Z} = 765.4$$

$$Q* = \overline{p}\cdot\overline{Z}* = (1,1,1) \begin{pmatrix} 305.5 \\ 326.9 \\ 322.0 \end{pmatrix} = 954.4$$

$$Q{*}{*} = \overline{p}\ \overline{Z}{*}{*} = (1,1,1) \begin{pmatrix} 194.4 \\ 214.9 \\ 201.6 \end{pmatrix} = 611.0$$

The direct cost and indirect cost may be calculated by

$$\text{Direct Cost} = \bar{p} \cdot \bar{Z}_D = \bar{p} \cdot C \cdot \bar{Y} = Q_D$$

$$\text{Indirect Cost} = \bar{p} \cdot \bar{Z}_I = \bar{p} \cdot D \cdot \bar{X} = Q_I$$

Substituting appropriate quantities into each of these equations for each case yields:

$$Q_D = (1,1,1) \begin{bmatrix} 82 \\ 127 \\ 72 \end{bmatrix} = 281.0$$

$$Q_I = (1,1,1) \begin{bmatrix} 143.0 \\ 138.1 \\ 183.3 \end{bmatrix} = 484.4$$

$$Q_{D*} = (1,1,1) \begin{bmatrix} 90.2 \\ 139.7 \\ 79.2 \end{bmatrix} = 309.1$$

$$Q_I{}^* = (1,1,1) \begin{bmatrix} 215.4 \\ 187.2 \\ 242.8 \end{bmatrix} = 645.4$$

$$Q_D{}^{**} = (1,1,1) \begin{bmatrix} 73.8 \\ 114.3 \\ 64.8 \end{bmatrix} = 252.9$$

$$Q_I{}^{**} = (1,1,1) \begin{bmatrix} 120.7 \\ 100.6 \\ 136.8 \end{bmatrix} = 358.1$$

In summary, the percentage change in direct costs, indirect costs, and total costs for each case is given as follows:

Case I. All model parameters increase by 10%.

$$\text{Percent Change in } Q = \frac{Q^* - Q}{Q} x100 = \frac{954.4 - 765.4}{765.4} x100$$

$$= 24.7\%$$

$$\text{Percent Change in } Q_D = \frac{Q^*_D - Q_D}{Q_D} x100 = \frac{309.1 - 281.0}{281.0} x100$$

$$= 10\%$$

$$\text{Percent Change in } Q_I = \frac{Q^*_I - Q_I}{Q_I} x100 = \frac{645.4 - 484.4}{484.4} x100$$

$$= 33.3\%$$

Case II. All model parameters decreased by 10%.

$$\text{Percent Change in } Q = \frac{Q^{**} - Q}{Q} x100 = \frac{611.0 - 765.4}{765.4} x100$$

$$= -20.1\%$$

$$\text{Percent Change in } Q_D = \frac{Q^{**}_D - Q_D}{Q_D} x100 = \frac{252.9 - 281.0}{281.0} x100$$

$$= - 10.0\%$$

$$\text{Percent Change in } Q_I = \frac{Q^{**}_I - Q_I}{Q_I} x100 = \frac{358.1 - 484.4}{484.4} x100$$

$$= - 26.5\%$$

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13. ABSTRACT

Analysis of the "Electric FYDP" force costing model's ability to estimate alternative force structure costs is presented and related to the current Department of Defense planning, programming, and budgeting system. The "Electric FYDP" cost model is described in economic and mathematical terms. An analysis of the model's workability, as defined by Michio Hatanaka, in modeling a military service economy for use in force cost prediction and force cost analysis is presented. The problems in estimating the parameters of the "Electric FYDP" are discussed and current parameter estimation procedures are appraised. Preliminary sensitivity analysis is performed. The conceptual use of estimates of force cost variance in scheduling procurement of future weapons systems with a binding budgetary constraint is discussed.

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